## DESIGN OF COMPRESSION MEMBER

**Step 1: Find approximate sectional area required.**

$$A_g\ (required) = \frac{P}{f_{cd}}$$

Where, P = Given Factored axial Load,

$f_{cd}$ = Assumed design compressive stress

- $f_{cd}$ can be found based on assumed slenderness ratio
- For I-section of column: Assume slenderness value $\frac{kL}{r_{min}} = 80\ to\ 150$ and $f_{cd}$ for buckling class b, can be found as $f_{cd}$ = 150 to 64 N/mm$^2$ *[table 9(b) page 41 (cl. 7.1.2.1) IS 800: 2007].* To start **Take $f_{cd}$ = 150 N/mm$^2$**
- For angle section of strut: Assume slenderness value $\frac{kL}{r_{min}} = 100\ to\ 150$ and $f_{cd}$ for buckling class b, can be found as $f_{cd}$ = 118 to 64 N/mm$^2$ *[table 9(b) page 41 (cl. 7.1.2.1) IS 800: 2007].* To start **Take $f_{cd}$ = 100 N/mm$^2$**

**Step 2: Select section from steel table based on gross area required. Note down the properties.**

**Step 3: Calculate slenderness ratio for selected cross section.**

$$\text{Slenderness ratio} = \frac{kL}{r_{min}}$$

Where, $r_{min}$ = minimum radius of gyration (for selected section from steel table

kL = effective length *(from table no. 11, page. 45, clause 7.2.2.)*

- Slenderness ratio should be less than 180 for column. *(Table 3 IS 800: 2007, pg.20)*

**Step 4: Calculate design compressive stress $f_{cd}$ for selected cross section.**

**(a) For Column**

**Method-1 Using equation** *(cl. no.7.1.2.1, IS 800:2007, page. 34)*

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \left[\phi^2 - \lambda^2\right]^{0.5}} = \chi\, f_y / \gamma_{m0} \leq f_y / \gamma_{m0}$$

Where, $\phi = 0.5[1+\alpha(\lambda - 0.2)+\lambda^2]$

$\lambda$ = non-dimensional effective slenderness ratio,

$= \sqrt{f_y / f_{cc}} = \sqrt{f_y \left(KL/r\right)^2 / \pi^2 E}$

$f_{cc}$ = Euler buckling stress $= \frac{\pi^2 E}{(KL/r)^2}$

α = imperfection factor *(page. 35 Table 7 IS 800: 2007)*

= buckling class of cross-section based on $h/b_f$ (Table no.10, page. – 44, cl.7.1.2.2)

$$\chi = \frac{1}{\varphi + (\varphi^2 - \lambda^2)^{0.5}}$$

**Method-2 Using tables**

- Select buckling class of cross-section based on $h/b_f$ (Table no.10, page. – 44, cl.7.1.2.2)
- Find out $f_{cd}$ based on slenderness ratio kL/r and $f_y$. (Table no. 9 a,b,c,d, page.40-43)

**(b) For strut** *(cl. no. 7.5.1.2, page no. 48, IS 800: 2007)*

$$f_{cd} = \frac{f_y / \gamma_{m0}}{\phi + \left[\phi^2 - \lambda_e^{\,2}\right]^{0.5}} \quad = \chi f_y / \gamma_{m0} \quad \leq f_y / \gamma_{m0}$$

Where, $\phi = 0.5[1+\alpha\,(\lambda_e - 0.2) + \lambda^2]$

$\lambda_e$ = equivalent slenderness ratio,

$$\lambda_e = \sqrt{k_1 + k_2\lambda_{vv}^2 + k_3\lambda_\phi^2}$$

$k_1$, $k_2$, $k_3$ = constants depending upon the end condition, as given in Table 12

$$\lambda_{vv} = \frac{\left(\dfrac{KL}{r_{vv}}\right)}{\varepsilon\sqrt{\dfrac{\pi^2 E}{250}}} \qquad \lambda_\phi = \frac{(b_1 + b_2)}{\varepsilon\sqrt{\dfrac{\pi^2 E}{250}} \times 2t}$$

*L* = laterally unsupported length of the member

$r_{vv}$ = radius of gyration about the minor axis

$b_1$, $b_2$ = width of the two legs of the angle

*t* = thickness of the leg

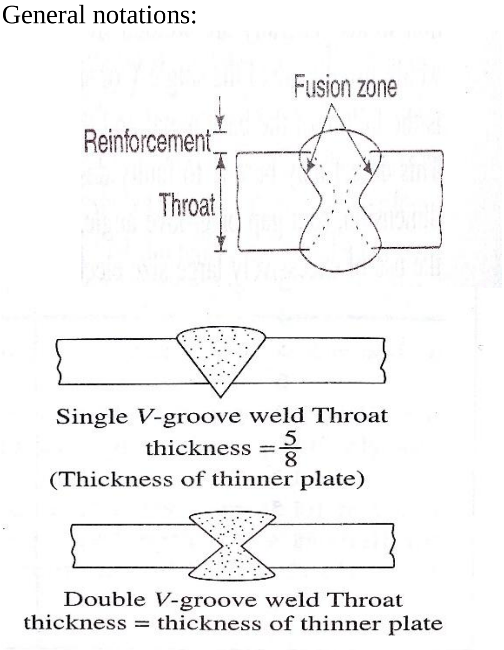
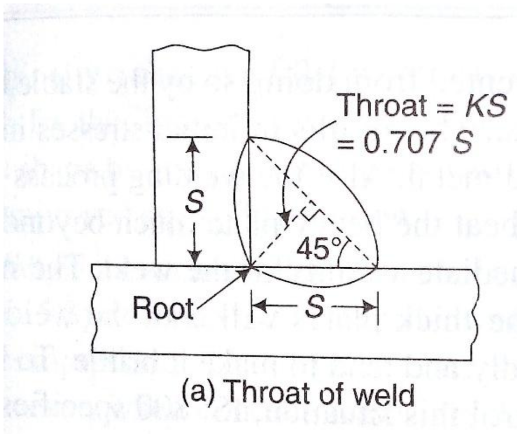
$\varepsilon$ = yield stress ratio $(250/f_y)^{0.5}$

**Step 5: Calculate design compressive strength.** *(cl. no.7.1.2, IS 800:2007, page. 34)*

$P_d = A_e f_{cd} <$ Given Loading P

Where, $A_e = A_g$ for plastic, compact and semi-compact

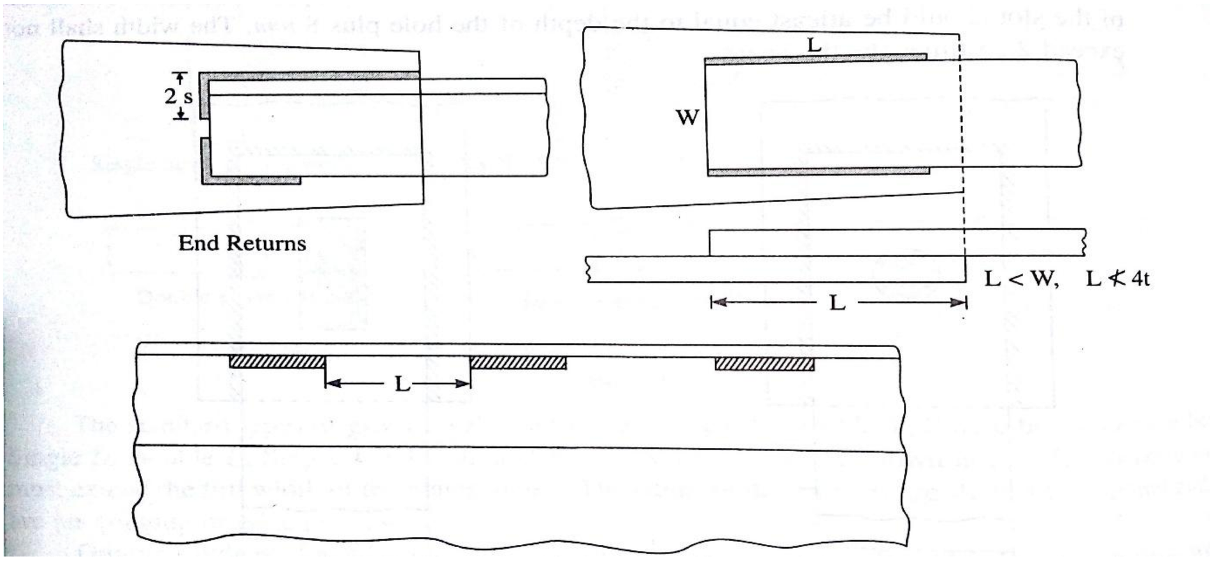
# Welded Connections

| Sr. No. | Groove or Butt Weld | Fillet Weld |
|---|---|---|
| 1. | Provided when members are in line | When members are perpendicular |
| 2. | Requires edge preparation | Edge preparation is not required. |
| 3. | Two same materials are connected, therefore serviceability condition is critical. | Due to stress concentration, ultimate strength criteria to be satisfied. |
| 4. | Smooth finish is possible. | Smooth finish is not possible. |
| 5. | General notations:<br><br> | General notations:<br><br> |
| 5. | Design Criteria: Direct Tension | Design Criteria: Shear |
| 6. | Design Strength:<br><br>(Strength = Area × stress )<br><br>$$Design\ Strength\ = \frac{l_w\, t_t f_y}{\gamma_{mw}}$$<br>*where,* $l_w$ *= length of weld*<br>$t_t$ *= throat thickness*<br>*= thickness of thinner plate for full penetration;*<br>*= 5/8 of thickness of thinner plate for partial penetration;*<br>$f_y$ *= yield stress*<br>$\gamma_{mw}$*= partial safety factor(table.5* | Design Strength: (Clause no. 10.5.7.1.1, pg.79)<br><br>(Strength = Area × stress )<br><br>$$Design\ Strength\ = \frac{l_w\, t_t f_{wd}}{\gamma_{mw}}$$<br>*Here,*<br>$f_{wd} = \frac{f_{wn}}{\gamma_{mw}}$ and $f_{wd} = \frac{f_u}{\sqrt{3}}$<br><br>*Hence*<br>$$Design\ Strength\ = \frac{l_w\, t_t f_u}{\sqrt{3}\,\gamma_{mw}}$$<br>*where,* $l_w$ *= length of weld*<br>$t_t$ *= throat thickness = ks*<br>*k = Table. 22 & Clause 10.5.3.2*<br>*= 0.707 normally taken*<br>*s = size of weld*<br>$f_u$*= smaller of ultimate stress of weld or parent material*<br>$\gamma_{mw}$*= partial safety factor.(Table.5)* |

**IS: Recommendation (IS 800: 2007)**

**End Returns:** *(Cl. No. 10.5.1.1, pg. 78)* Length of end returns should not be less than 2*s*

**Lap Joints:** *(Cl. No. 10.5.1.2, pg. 78)*
In lap connections, the minimum length of weld should not be less than 4 times the thickness of the thinner part joined or 40mm whichever is more. If only side fillets are used, the length of the weld on either edge should not be less than the transverse spacing between the welds.




**For intermittent fillet welds:** *(Cl. No. 10.5.5.2, pg. 79)*

Effective length (wl) ≥ 4*s* or 40 mm, whichever is greater
Clear spacing (uwl) ≥ 12*t* (for compression)
≤ 16*t* (for tension)
≤ 200 mm
Where *t* is the thickness of thinner part joined

**Size of weld:** (cl. no. 10.5.2.3, pg. no.78 and Table. 21):

Fillet weld shall not be less than 3mm.

| Thickness (*t*) of thicker part in mm | S (mm) |
|---|---|
| $t \leq 10$ | 3 |
| $10 < t \leq 20$ | 5 |
| $20 < t \leq 32$ | 6 |
| $32 < t \leq 50$ | 10 (8 mm for first run) |
| $t > 50$ | Special precaution like pre-heating to be taken |

**Throat thickness:** (cl. no. 10.5.3.2- Table. 22, pg. no. 78)

**Max. size of weld:** (cl. no. 10.5.8.1, pg. no. 79)

Maximum size of fillet weld = thickness of thinner plate – 1.5

# Bolted Connections

## Introductions:

- Connections are always needed to connect two members.
- It is necessary to ensure functionality and compactness of structures.
- Prime role of connections is to transmit force from one component to another.
- Steel connections can be made by bolts or welds.
- Connections accounts for more than half cost of steel structure.
- Connections are designed more conservative than members because they are more complex.

## Types of Bolts

- Unfinished Bolt – ordinary, common, rough or black bolts
- High strength Bolt – friction type bolts

## Classifications of Bolted connections:

Based on Joint:

- Lap Joint
- Butt Joint

Based on Load transfer Mechanism**:**

- Shear and Bearing,
- Friction

## Grade classification of Bolts:

- The grade classification of a bolt is indicative of the strength of the material of the bolt. The two grade of bolts commonly used are grade 4.6 and 8.8.
- For 4.6 grade 4 indicates that ultimate tensile strength of bolt = 4 × 100 = 400 N/mm$^2$ and 0.6 indicates that the yield strength of the bolt is 0.6 × ultimate strength = 0.6 × 400 = 240 N/mm$^2$

| Grade of bolt | 4.6 | 5.6 | 6.5 | 6.8 | 8.8 |
|---|---|---|---|---|---|
| $F_{yb}$( N/mm$^2$) | 240 | 300 | 300 | 480 | 640 |
| $F_{ub}$( N/mm$^2$) | 400 | 500 | 600 | 800 | 800 |

Properties of High Strength Friction Grip Bolts

## Common Terms in Bolted Connections:

- **Gauge Line:** This is a line parallel to the direction of stress (or load) along which the bolts are placed.
- **Pitch:** It is the distance between the centres of two consecutive bolts measured along a row of bolts (Gauge Line). It is denoted by *p*.
- **Gauge Distance:** This is the perpendicular distance between adjacent gauge lines. It is denoted by *g*.
- **Edge Distance:** This is the shortest distance from the edge of the member to the extreme bolt hole centre along the Gauge line.
- **End Distance:** It is the shortest distance from the edge of the member to the extreme bolt hole centre perpendicular to the Gauge line.

## IS 800: 2007 Provisions:

**Clearance for Holes for Fastener (Bolts)** (IS 800:2007 Clause 10.2.1, page no. 73)

- The diameter of the hole should be the nominal diameter of the bolt plus the clearance as given below:

Standard Clearance for Fastener Holes
(IS 800: 2007, Table no. 19)

| Nominal size of fastener, d mm | Size of the hole = Nominal diameter of the fastener + Clearance | | | |
|---|---|---|---|---|
| | Standard clearance in diameter and width of slot mm | Over size clearance in diameter | Clearance in the length of the slot | |
| | | | Short slot mm | Long slot mm |
| 12-14 | 1.0 | 3.0 | 4.0 | 2.5d |
| 16-24 | 2.0 | 4.0 | 6.0 | 2.5d |
| 24 | 2.0 | 6.0 | 8.0 | 2.5d |
| More than 24 | 3.0 | 8.0 | 10.0 | 2.5d |

**Minimum pitch:** *(IS 800:2007, Clause 10.2.2, page no. 73)*
The distance between the centres of the bolts in the direction of stress should not be less than 2.5 times the nominal diameter of the bolt

**Maximum pitch** *(IS 800:2007, Clause 10.2.3, page no. 74)*

- 32*t* or 300 mm, whichever is less for the bolts in members including the tacking bolts. *(Clause 10.2.3.1, page no. 74)*

- $16t$ or 200 mm, whichever is less for the bolts in tension members, Where t is the thickness of thinner plate *(Clause 10.2.3.2, page no. 74)*
- 12t or 200 mm, whichever is less for the bolts in compression members , Where t is the thickness of thinner plate *(Clause 10.2.3.2, page no. 74)*

**Edge and End distance** *(IS 800:2007, Clause 10.2.4.2, page no. 74)*

- **Minimum edge and end distances** from the centre of any hole to the nearest edge of a plate should not be less than 1.7 times the hole diameter for sheared or hand- flame cut edges; and 1.5 times the hole diameter for rolled, machine-flame cut, sawn and planned edges

- **Maximum edge distance** from the centre of hole to the nearest edge should not exceed $12t\varepsilon$, where $\varepsilon$ = sqrt(250/$f_y$) and $t$ is the thickness of the thinner outer plate.

**Tacking Bolts** *(IS 800:2007, Clause 10.2.5.2, page no. 74)*

- These are the additional bolts provided other than strength consideration.
- The maximum pitch of these bolts should be $32t$ or 300 mm, whichever is less, where $t$ is the thickness of the thinner plate.
- If the members are exposed to weather, the pitch should not exceed 16 times the thickness of the outside plate or 200 mm, whichever is less.

**Strength Provisions:**

The design Strength $V_{db}$ of bolt is lesser of
1. Design shear strength $V_{dsb}$ of bolt, and
2. Design bearing strength $V_{dpb}$ of bolt.

- **Shear Capacity of a Bolt** *(IS 800:2007, Clause 10.3.3, page no. 75)*

The design strength of a bolt in shear $V_{dsb}$ is given by

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{fu}{\sqrt{3}} \times [n_n A_{nb} + n_s A_{sb}]$$

Where,
$V_{nsb}$ = nominal shear capacity of a bolt
$f_u$ = Ultimate tensile strength of the bolt material
$n_n$ = Number of shear planes with threads intercepting the shear plane
$n_s$ = Number of shear planes without threads intercepting the shear plane.

$A_{sb}$ =Nominal plain shank area of the bolt
$A_{nb}$ = Net shear area of the bolt at threads
= Area corresponding to root diameter at the thread
= $\frac{\pi}{4}[d - 0.9382p]^2$ (I.S. 1367 part-1)
= 0.78 to 0.80 of gross area of shank (see table)
$\gamma_{mb}$ = Partial safety Factor of safety for the bolt material =1.25 *(IS 800:2007, Table 5, page no. 30)*

- **Bearing Capacity of a Bolt** *(IS 800:2007, Clause 10.3.4, page no. 75)*

The design strength of a bolt in bearing $V_{dpb}$ is given by

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$$V_{npb} = 2.5k_b\, dtf_u$$

Where,
$V_{npb}$ = nominal bearing strength of a bolt
$k_b$ = smallest of $\frac{e}{3d0}$ , $\frac{p}{3d_0}$ – 0.25, $\frac{f_{ub}}{f_u}$, 1
*e, p* = end and pitch distances of the fastener along bearing direction
$d_o$ = diameter of hole
*d* = nominal diameter of the bolt
*t* = summation of the thickness of the connected plates experiencing bearing stress in the same direction, or if the bolts are counter sunk, the thickness of the plate minus one half of the depth of counter sinking .

- **Tension Capacity of a Bolt** *(IS 800:2007, Clause 10.3.5, page no. 76)*

The design strength of a bolt in tension $T_{db}$

$$Tdb = \frac{T_{nb}}{\gamma_{mb}}$$

$$T_{nb} = 0.9\, f_{ub}\, A_{nb} < f_{yb} A_{sh} \frac{\gamma_{mb}}{\gamma_{mo}}$$

Where,
$T_{nb}$ = Nominal tensile capacity of the bolt, calculated as,
$f_{ub}$ = yield stress of the bolt
$A_n$ = Net tensile stress area = area at bottom of the threads, and
$A_{sh}$ = Shank area of the bolt
$\gamma_{mo}$ = Partial safety factor for resistance governed by yielding = 1.10
$\gamma_{mb}$ = Partial safety factor for material of bolt

- **Bolt subjected to Combined Shear and Torsion** *(IS 800:2007, Clause 10.3.6, page no. 76)*

A bolt subjected to shear and torsion simultaneously should satisfy the condition

$$\left(\frac{V_{sb}}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1$$

Where,
$V_{sb}$ = Factored shear force acting on the bolt
$V_{db}$ = Design shear capacity of the bolt
$T_b$ = Factored tensile force acting on the bolt
$T_{db}$ = Design tensile capacity of the bolt

- **Bolt subjected to Friction** *(IS 800:2007, Clause 10.4.3, page no. 76)*

$$V_{dsf} = \frac{V_{nsf}}{\gamma_{mf}}$$

$$V_{nsf} = \mu_f n_e K_h F_o$$

Where,
$\mu_f$ = coefficient of friction (slip factor) as specified in Table 20 ($\mu_f$ = 0.55
$n_e$ = number of effective interfaces offering frictional resistance to slip
$K_h$ = 1.0 for fasteners in clearance holes,
= 0.85 for fasteners in oversized and short slotted holes loaded perpendicular to the slot.
$\gamma_{mf}$ = 1.10 (if slip resistance is designed at service load
= 1.25 (if slip resistance is designed at ultimate load
$F_o$ = minimum bolt tension (proof load) at installation and may be taken as $A_{nb} f_o$
$A_{nb}$ = Net shear area of the bolt at threads
$f_o$ = proof stress ( = $0.70 f_{ub}$ )

## Load Transmission by Different Types of Bolts

- **Load transmitted by Black Bolt (bearing type**)
Due to load the plates slip so as to come into contact with edges of the hole. The load transmission is by baring of the bolt and shear on the shank.
- **Load transmission by High Strength friction grip Bolt**

In this case tightening the nut tension T is transmitted to the bolt to reach tensile stress equal to 0.8 to 0.9 times the yield stress.

As a consequence of this initial tension in the bolt, the two plates get tightly clamped together.

If the tensile force in the bolt is T, a compressive force T becomes the clamping force.

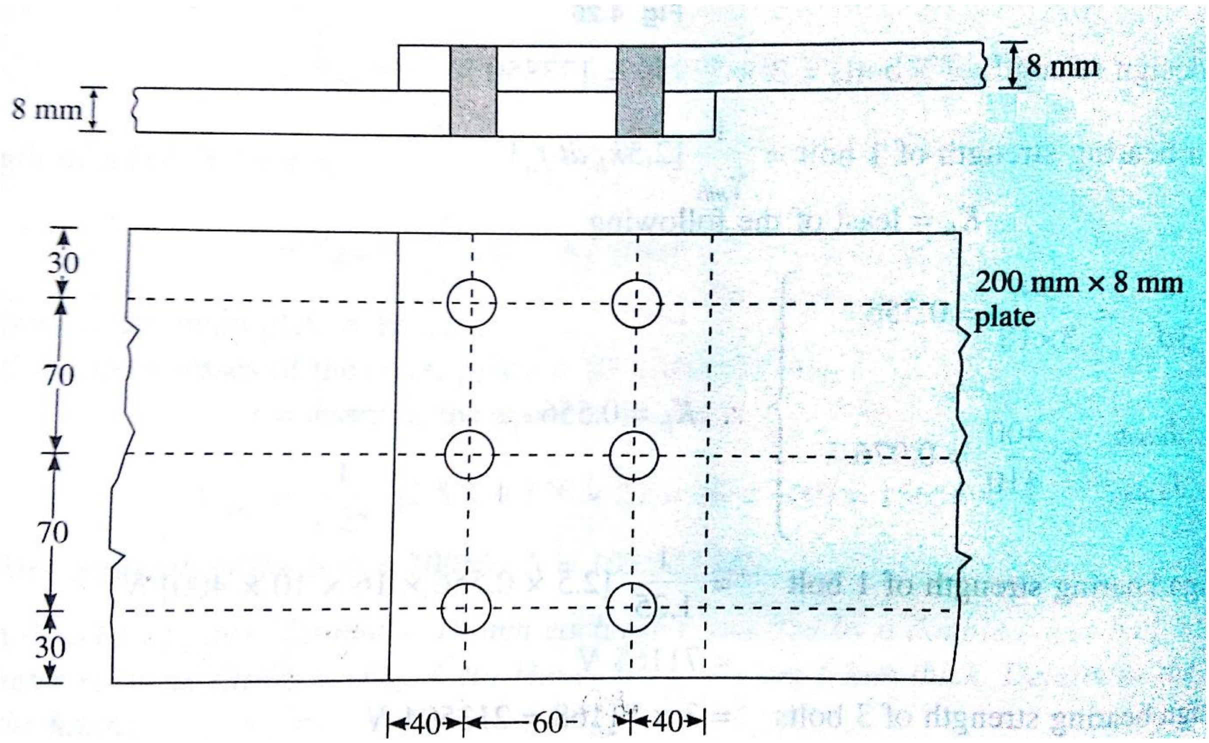
Plates are prevented from slipping due to frictional resistance F whose value is $\mu T$, where $\mu$ is coefficient of friction between the plate surfaces.

If pull P applied to the plates is less than $\mu T$, plates do not slip at all and the load transmission from one plate to other place entirely by frictional resistance only.

IF the applied force P exceed $\mu T$, slip between the plate occurs resulting in shearing and bearing stresses in the bolt.

| Strength | Black bolt or Bearing Type bolt | HSFG Bolt | |
|---|---|---|---|
| | | Slip Resistance | Non-slip resistance |
| Shear Strength | √ | | √ |
| Bearing Strength | √ | | √ |
| Tensile Strength | | √ | |
| Frictional Strength | | √ | |

**Example:1** Two plates 200mm × 8mm are to be connected by 16mm diameter bolts in a lap joint. Calculate the strengths for the black type (Bearing type bolts). Take 4.6 grade of bolts. Take ultimate tensile strength of plate = 410 $N/mm^2$.




Given Data: d = 16mm

$d_o$= 16 + 2 = 18 mm (Table 19, Page no. 19, Page no. 73)

t = thickness of plate = 8 mm

- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{fu}{\sqrt{3}} \times [n_n A_{nb} + n_s A_{sb}]$$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \times \left[\left(1 \times 0.78 \times \frac{\pi}{4} \times 16^2\right) + \left(0 \times \frac{\pi}{4} \times 16^2\right)\right]$$

= *28.97 kN*

- **Bearing Capacity**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$V_{npb} = 2.5 k_b \, d t f_u$

$$V_{dpb} = \frac{2.5 \times 0.741 \times 16 \times 8 \times 410}{1.25}$$

$K_b$ *is least of*
$e/3d_o = 40/(3 \times 18) = 0.741;$
$p/3d_o - 0.25 = [60/(3 \times 18)] - 0.25 = 0.86;$
$f_{ub}/f_u = 400/410 = 0.976;$
*1*

$= 77.77\ kN$

**Example:2** Determine the design strength of 22mm dia. Bolt for the cases given below.
(1). Lap Joint
(2) Single cover butt joint with 12mm cover plate.
(3) Double cover butt joint with 10mm cover plate.
Consider main plate is 16mm thick. Take 4.6 grade of bolts.
Take ultimate tensile strength of plate = 410 $N/mm^2$.

Given Data: d = 22mm
$d_o$= 22 + 2 = 24 mm (Table 19, Page no. 19, Page no. 73)
t = thickness of plate = 16 mm

**Case 1:**

- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{fu}{\sqrt{3}} \times [n_n A_{nb} + n_s A_{sb}]$$

$$V_{dsb} = \frac{400}{\sqrt{3}\times 1.25} \times \left[\left(1 \times 0.78 \times \frac{\pi}{4} \times 22^2\right) + \left(0 \times \frac{\pi}{4} \times 22^2\right)\right]$$

$= 54.78\ kN$

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$$V_{npb} = 2.5 k_b\, d t f_u$$

$$V_{dpb} = \frac{2.5 \times 0.556 \times 22 \times 16 \times 410}{1.25}$$

$= 160.48\ kN$

- $e = 1.5\ d_o = 1.5 \times 24 = 36 \approx 40mm$
- $p = 2.5d = 2.5 \times 22 = 55 \approx 60\ mm$
- $K_b$ *is least of*
  - $e/3d_o = 40/(3 \times 24) = 0.556;$
  - $p/3d_o - 0.25 = [60/(3 \times 24)] - 0.25 = 0.58;$
  - $f_{ub}/f_u = 400/410 = 0.976;$
  - *1*
- $t = 16mm$
- $d = 22mm$

**Case 2:** Single cover butt joint with 12mm cover plate

- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = 54.78$$

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{2.5 \times 0.556 \times 22 \times 12 \times 410}{1.25}$$
$$= 120.36\ kN$$

**Case 3:** Double cover butt joint with 10mm cover plate.

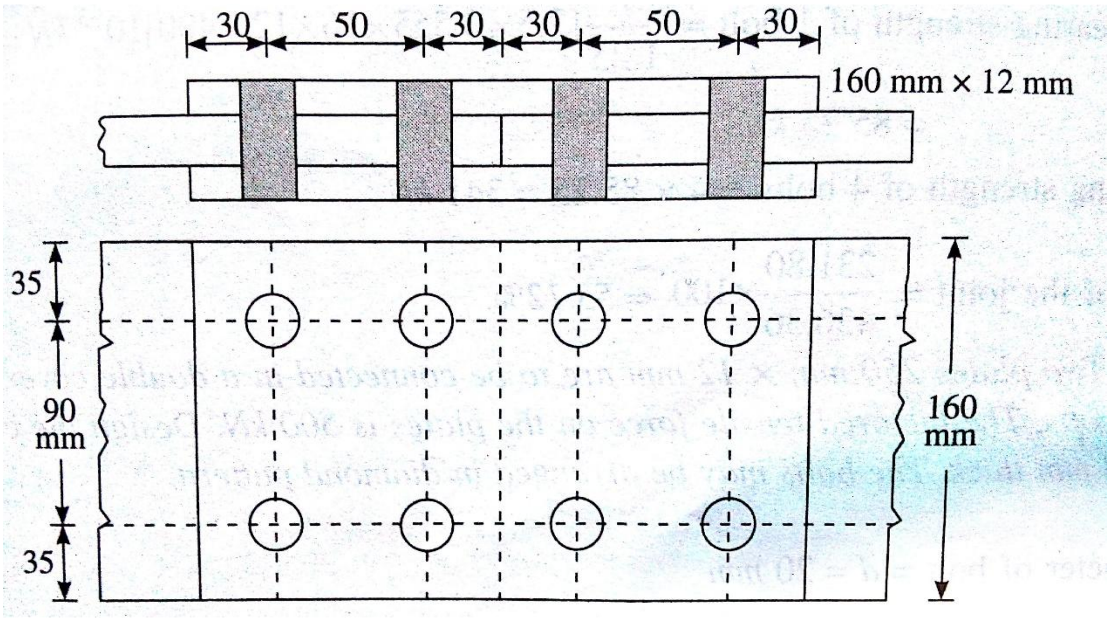
- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = 2 \times 54.78 = 109.56\ kN$$

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{2.5 \times 0.556 \times 22 \times 16 \times 410}{1.25}$$
$$= 160.48\ kN$$

**Example 3**: Find the efficiency of the butt join shown in figure. Bolts are 16mm diameter of grade 4.6. Cover plates are 8mm thick.




- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{fu}{\sqrt{3}} \times [n_n A_{nb} + n_s A_{sb}]$$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \times \left[\left(2 \times 0.78 \times \frac{\pi}{4} \times 16^2\right) + \left(0 \times \frac{\pi}{4} \times 16^2\right)\right]$$

$= 57.95\ kN$

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$$V_{npb} = 2.5 k_b\, d t f_u$$

$$V_{dpb} = \frac{2.5 \times 0.56 \times 16 \times 12 \times 410}{1.25}$$

$= 88.16\ kN$

- *e = 30mm*
- *p = 50 mm*
- $K_b$ *is least of*
  - $e/3d_o = 30/(3 \times 18) = 0.56$;
  - $p/3d_o - 0.25 = [50/(3 \times 18)] - 0.25 = 0.67$;
  - $f_{ub}/f_u = 400/410 = 0.976$;
  - *1*
- *t = 16mm*
- *d = 12mm*

$$Efficiency\ of\ the\ joint = \left[\frac{Strength\ of\ the\ Joint\ per\ pitch\ length}{Strength\ of\ solid\ plate\ per\ pitch\ Length}\right] \times 100$$

- Strength the joint per pitch length = 57.95 kN
- $Strength\ of\ solid\ plate\ per\ pitch\ length = \frac{0.9 A_n f_u}{\gamma_{m1}}$ ( clause no. 6.3.1, page no. 32,IS 800:2007)

$$= \frac{0.9 \times (50 - 16) \times 12 \times 410}{1.25}$$

= *120.44 kN*

$$Efficiency\ of\ the\ joint\ = \left[\frac{57.95}{120.44}\right] \times 100$$

= *48.11 %*

**Example:** An ISA 100mm × 100mm ×10mm carries a factored tensile load of 100kN.It is to jointed with a 12mm thick gusset plate. Design high strength bolted joint when,

(a) no slip is permitted.

(b) when slip is permitted.

Take bolt grade =8.8 with 16mm dia.

Solution:

$f_{ub}$ = 800 N/mm² for 8.8 grade.

**(a) Slip critical connection:**

- **Frictional Strenght** *(IS 800:2007, Clause 10.4.3, page no. 76)*

$$V_{dsf} = \frac{V_{nsf}}{\gamma_{mf}}$$

$$V_{nsf} = \mu_f\, n_e\, K_h\, F_o$$

Where,

$\mu_f$ = coefficient of friction (slip factor) as specified in Table 20 ($\mu_f$ = 0.55) = 0.55

$n_e$ = number of effective interfaces offering frictional resistance to slip = 1

$K_h$ = 1.0 for fasteners in clearance holes,
= 0.85 for fasteners in oversized and short slotted holes loaded perpendicular to the slot.

$\gamma_{mf}$ = 1.10 (if slip resistance is designed at service load
= 1.25 (if slip resistance is designed at ultimate load

$A_{nb}$ = Net shear area of the bolt at threads = $\left(0.78 \times \frac{\pi}{4} \times 16^2\right) = 157mm^2$

$f_o$ = proof stress ( = $0.70f_{ub}$) = 0.70 × 800 = 560 N/mm²

$F_o$ = minimum bolt tension (proof load) at installation and may be taken as $A_{nb}\, f_o$ = *560 × 0.157 = 87. 9 kN*

$$V_{dsf} = \frac{\mu f \; ne \; Kh \; Fo}{\gamma_{mf}}$$

$$V_{dsf} = \frac{0.55 \times 1 \times 1 \times 87.9}{1.25}$$

= 38.65 kN

- **Tension Capacity of a Bolt** *(IS 800:2007, Clause 10.3.5, page no. 76)*

The design strength of a bolt in tension $T_{db}$

$$Tdb = \frac{T_{nb}}{\gamma_{mb}}$$

$$T_{nb} = 0.9 \, f_{ub} \, A_{nb}$$

$$Tdb = \frac{0.9 \, f_{ub} \, A_{nb}}{\gamma_{mb}} = \frac{0.9 \times 800 \times 157}{1.25} = 90.32 \, kN$$

Bolt value = 38. 65 kN

$$No. of \; bolt = \frac{100}{38.65} \cong 3$$

**(a) Slip allowed connection:**

- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{fu}{\sqrt{3}} \times [n_n A_{nb} + n_s A_{sb}]$$

$$V_{dsb} = \frac{800}{\sqrt{3} \times 1.25} \times \left[\left(1 \times 0.78 \times \frac{\pi}{4} \times 16^2\right) + \left(0 \times \frac{\pi}{4} \times 16^2\right)\right]$$

= *58 kN*

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$V_{npb} = 2.5k_b\ dtf_u$

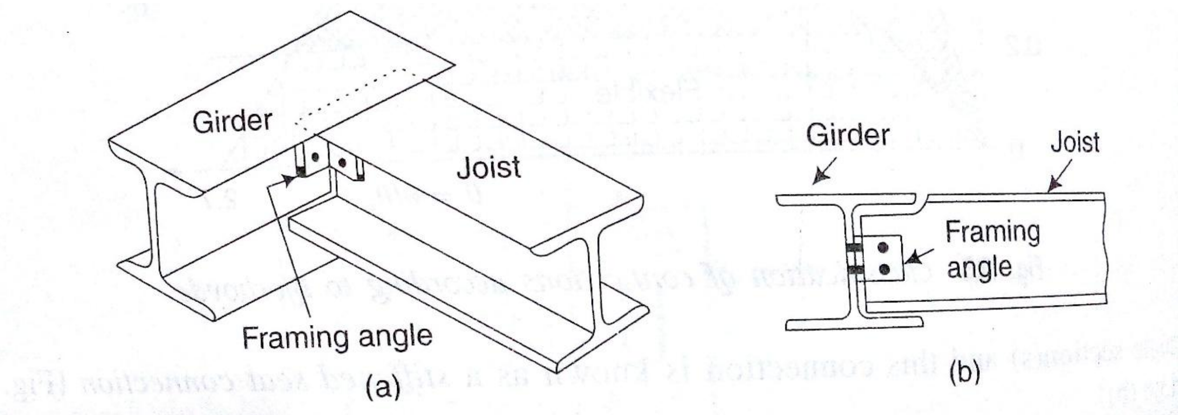
$$V_{dpb} = \frac{2.5 \times 0.56 \times 16 \times 10 \times 410}{1.25}$$

$= 73.47\ kN$

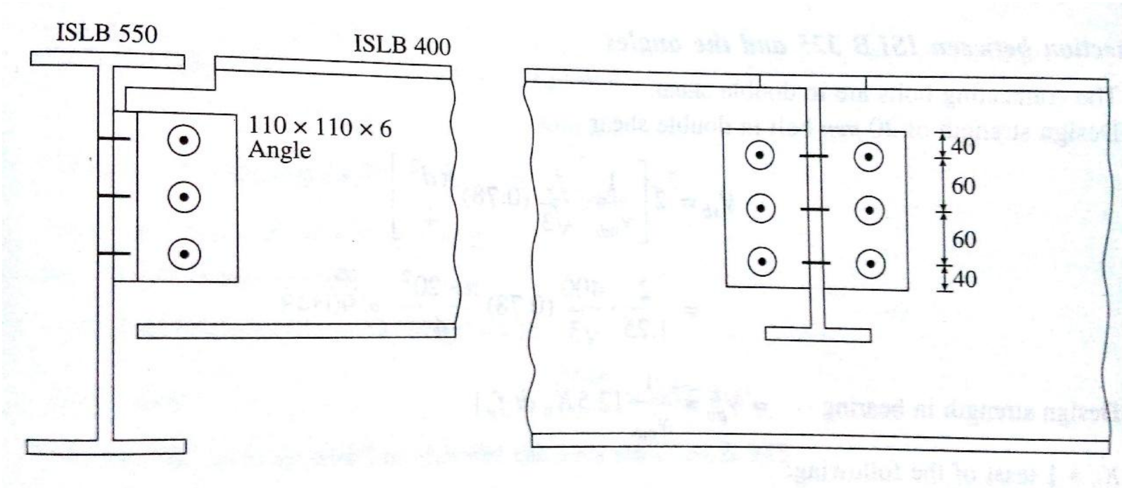
- $e = 30mm$
- $p = 50\ mm$
- $K_b$ *is least of*
  - $e/3d_o = 30/(3 \times 18) = 0.56;$
  - $p/3d_o - 0.25 = [50/(3 \times 18)] - 0.25 = 0.67;$
  - $f_{ub}/f_u = 800/410 =;$
  - *1*
- $t = 10mm$
- $d = 16mm$

Bolt value = 58 kN

$$No. of\ bolt = \frac{100}{58} = 1.72 \cong 2$$

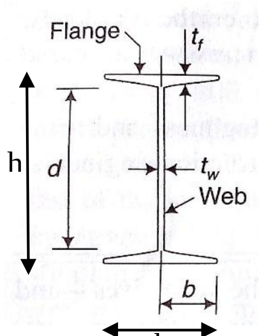



**Example:** An I beam ISLB400@558.2 N/m is to be connected to the web of a main beam ISLB550@846.6 N/m. The factor reaction of 220 kN. Design the connection if the cleat angel size is 110 × 110 × 6 as shown in figure. Use 20mm diameter bolts.




Properties from steel table:

| Properties | Notation | ISLB400 | ISLB550 |
|---|---|---|---|
| Height | h | 400 mm | 550 mm |
| Width of flange | $b_f$ | 165mm | 190mm |
| Thickness of flange | $t_f$ | 12.5mm | 15mm |
| Thickness of web | $t_w$ | 8mm | 9.9mm |




**Connection between the angles and the web of the secondary beam:**
The connecting bolts are in double shear:

- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

$$V_{dsb} = \frac{V_{nsb}}{\gamma_{mb}}$$

$$V_{nsb} = \frac{fu}{\sqrt{3}} \times [n_n A_{nb} + n_s A_{sb}]$$

$$V_{dsb} = \frac{400}{\sqrt{3} \times 1.25} \times \left[\left(2 \times 0.78 \times \frac{\pi}{4} \times 20^2\right) + \left(0 \times \frac{\pi}{4} \times 16^2\right)\right]$$

= *90.54 kN*

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$$V_{npb} = 2.5k_b\, dtf_u$$

$$V_{dpb} = \frac{2.5 \times 0.606 \times 20 \times 8 \times 410}{1.25}$$

$= 79.51\ kN$

> - $e = 1.5\ d_o = 1.5 \times 22 = 33 \approx 40mm$
> - $p = 2.5d = 2.5 \times 20 = 50 \approx 60\ mm$
> - $K_b$ *is least of*
>   - $e/3d_o = 40/(3 \times 22) = 0.606;$
>   - $p/3d_o - 0.25 = [60/(3 \times 20)] - 0.25 = 0.659;$
>   - $f_{ub}/f_u = 400/410 = 0.976;$
>   - *1*
>
> - $t = 16mm$
> - $d = 22mm$

Bolt value = 79.51 kN

$$Number\ of\ bolts\ required = \frac{220}{79.51} = 2.78 \cong 3\ bolts$$

**Connection between the angles and the web of the main beam:**
The connecting bolts are in single shear:

- **Shear Strength** (*IS 800:2007, Clause 10.3.3, page no. 75)*

  Design strength of the bolt in single shear = 45.27 kN

- **Bearing Strength**:*(IS 800:2007, Clause 10.3.4, page no. 75)*

$$V_{dpb} = \frac{V_{npb}}{\gamma_{mb}}$$

$$V_{npb} = 2.5k_b\, dtf_u$$

$$V_{dpb} = \frac{2.5 \times 0.606 \times 20 \times 9.9 \times 410}{1.25}$$

= 98.39 kN

Bolt value = 45.27 kN

$$Number\ of\ bolts\ required = \frac{220}{45.27} = 4.85 \cong 5\ bolts$$

# Tension Member

## Design Steps for Tension Member:

**Step-1** Calculate gross area required $A_g$ to carry the designed load $T$ is obtained by the equation. *(cl. no. 6.2, IS 800:2007, pg. no. 32)*

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}} \quad \Rightarrow \quad A_g = \frac{T_{dg}\,\gamma_{m0}}{f_y}$$

Where, $A_g$ = gross area of cross section
$\gamma_{mo}$ = partial safety factor for failure by yielding *(see Table5,page.30).*
$f_y$ = yield stress of material = 250 N/mm$^2$

**Step-2** Select section from steel table based on gross area required. Note down the properties.

**Step-3** Check three strengths of selected cross section under tensile load.

(a) Gross section Yielding
Check gross section yielding strength from following equation *(cl. no. 6.2, IS 800:2007, pg. no. 32)*

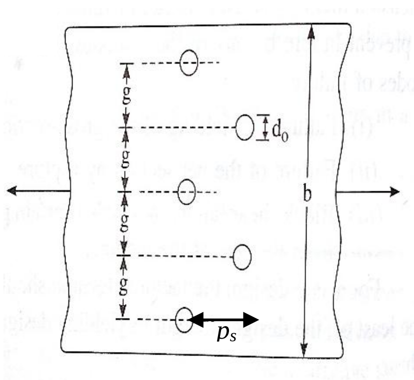
$T_{dg} \geq$ Given Loading

$$T_{dg} = \frac{A_g f_y}{\gamma_{m0}}$$

(b) Net section rupture
**Plates:**
Check net section rupture strength from following equation *(cl. no. 6.3.1, IS 800:2007, pg. no. 32)*

$T_{dn} \geq$ Given Loading

$$T_{dn} = \frac{0.9 A_n f_u}{\gamma_{m1}}$$




Where, $A_n$ = net effective c/s area of the member.
$A_n$ = [b – n$d_0$ + Σ($p_s^2$/4g)] × t
$b, t$ = width and thickness of plate respectively
$d_0$ = diameter of bolt hole
$g$ = gauge length between the bolt holes, as shown in figure,
$p_s$ = staggered pitch as shown in figure
$\gamma_{m1}$ = partial safety factor for failure at ultimate stress = 1.25 *(see Table5, page.30).*
$f_u$ = ultimate stress of material = 410 N/mm$^2$

**Angles:** *(cl. no. 6.3.3, IS 800:2007, pg. no. 33)*

$$T_{dn} = \frac{0.9A_{nc}f_u}{\gamma_{m1}} + \frac{\beta A_{go}f_y}{\gamma_{mo}}$$

Where,

$$\beta = 1.4 - 0.076\left[\frac{w}{t}\right]\left[\frac{f_y}{f_u}\right]\left[\frac{b_s}{L_c}\right] \le 0.9\frac{f_u\,\gamma_{mo}}{f_y\,\gamma_{m1}} \ge 0.7$$

$w$= Outstand leg width

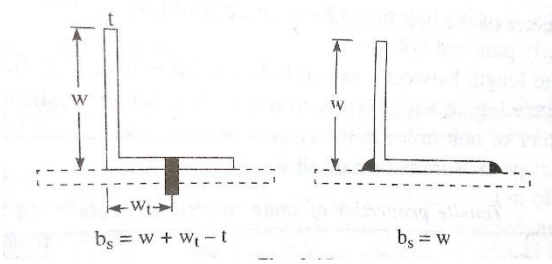
$L_c$ = *Length of the end connection, distance between the outer most blots in the end joint measured along the load direction or the length of weld along the load direction.*

$b_s$ = *shear lag width as shown in figure.*

$A_{nc}$ = *net area of connected leg,*

$A_{go}$ = *gross area of outstanding leg,*

$t$ = *thickness of leg*




(c) Block shear Failure

Check block shear failure strength from following equation

$T_{db} \ge$ Given Loading

Block shear failure $T_{db}$ = smaller of following *(cl. no. 6.4.1, IS 800:2007, pg. no. 33)*

$$T_{db} = \frac{A_{vg}f_y}{\sqrt{3}\gamma_{m0}} + \frac{0.9A_{tn}f_u}{\gamma_{m1}}$$ *(Against shear yield and tensile fracture)*

**or**

$$T_{db} = \frac{0.9A_{vn}f_u}{\sqrt{3}\gamma_{m1}} + \frac{A_{tg}f_y}{\gamma_{m0}}$$ *(Against shear fracture and tensile yielding)*

Where,

$A_{vg}$, $A_{vn}$ = Minimum gross and net area in shear along bolt line parallel to external force respectively.

$A_{tg}$, $A_{tn}$ = Minimum gross and net area in tension from the bolt hole to the toe of the angle end bolt line perpendicular to the line of force respectively.

$f_u$,$f_y$ = ultimate and yield stress of the material respectively = 410 and 250 $N/mm^2$

**Step 4:** Slenderness ratio should be less than 180. *(Table 3 IS 800: 2007, pg.20)*

$$Slenderness\ ratio = \frac{l}{r_{min}}$$

$$Radius\ of\ gyration = \sqrt{\frac{I}{y}}$$

$r_{min}$ = minimum radius of gyration.

I = moment of inertial

y = distance of extreme fibre from neutral axis.

# DESIGN OF BEAM

## LATERALLY SUPPORTED:

**STEP 1: FIND OUT ULTIMATE LOAD ON BEAM.**

Factored Ultimate Load (Factored Load) w = 1.5 × Working Load

**STEP 2: FIND OUT MAXIMUM BENDING MOMENT (M) AND SHEAR FORCE (V) ON BEAM.**

**STEP 3: CALCULATE PLASTIC SECTION MODULUS REQURIED FOR TRIAL SECTION.**

$$Z_{P(required)} = \frac{M\gamma_O}{f_y}$$

**STEP 4: SELECT SUITABLE SECTION BASED ON $Z_p$ FROM IS: 800: 2007, PAGE NO. 138, 139. WRITE DOWN SE TIONAL PROPERTIES.**

**STEP 5: SECTION CLASSIFICATION.**

a. Find out value of $b/t_f$ and $d/t_w$. (refer Figure. 2, Page no. 19, IS 800: 2007 to find b and d)

$t_f$ = thickness of flange $t_w$ = thickness of web.

b. Refer Table 2, Page no. 18, IS 800: 2007 and classify the section semi-compact, compact, plastic or slender.

**STEP 6: CHECK FOR SHEAR.** (Clause no. 8.4.1., Page no. 59, IS 800: 2007)

**a. Find out design shear strength $V_d$.**

$$\boldsymbol{V_d = \frac{f_y}{\sqrt{3\gamma_{mo}}} h t_w}$$

**b. Beam is checked for high / low shear case**

$V \leq 0.6\ V_d$ low shear case

$V > 0.6\ V_d$ high shear case

**STEP 6: CHECK FOR BENDING.**

**a. For low shear Case** (Clause no. 8.2.1.2, Page no. 53, IS 800: 2007)

$M_d > M$

$M_d$ = Design Bending Strength

M = Bending Moment

$$M_d = \beta_b Z_p \frac{f_y}{\gamma_{mo}} \leq 1.2 Z_e \frac{f_y}{\gamma_{mo}}\ (for\ simply\ supported\ beam)$$

$$\leq 1.5 Z_e \frac{f_y}{\gamma_{mo}}\ (for\ cantilever\ beam)$$

$\beta_b$ = 1 for plastic and compact sections.

= $Z_e/Z_p$ for semi compact sections.

$Z_e$ = Elastic section Modulus

$Z_p$ = Plastic section Modulus

**b. For High shear Case** (Clause no. 8.2.1.3, Page no. 53, IS 800: 2007)

Refer Clause no. 8.2.1.3, Page no. 53, IS 800: 2007. Generally low shear case is preferred.

**STEP 7: CHECK FOR WEB BUCKLING AT SUPPORT** (Clause no. 8.7.3.1, Page no. 67, IS 800: 2007)

a. Capacity of section = $A_b f_{cd} > V$

b. $A_b = (b_1 + n_1)\ t_w$ when load is at support

$A_b = (b_1 + 2n_1)\ t_w$ when load is not at support

Where, $b_1$= stiff bearing length of load = assume between 0 to 100mm

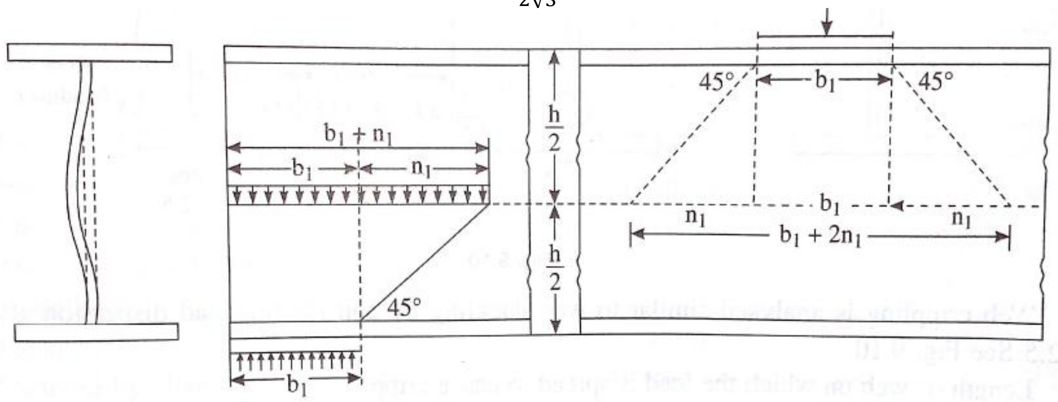
$n_1$ = for 45˚ dispersion consider h/2

d. Find out $F_{cd}$ = Design Compressive Stress considering class c and $f_y$ = 250 MPa.

Slenderness ratio $= \frac{kl}{r} = \frac{0.7d}{r}$

D = depth of the web between the flanges

r = least radius of gyration of the section = $\frac{t_w}{2\sqrt{3}}$



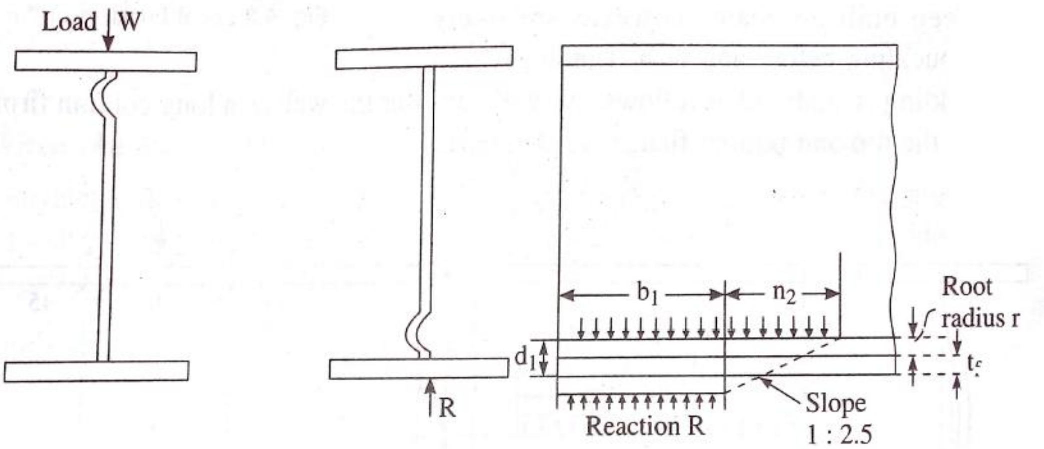

**STEP 7: CHECK FOR WEB CRIPPLING** (Clause no. 8.7.4, Page no. 67, IS 800: 2007)

Design crippling strength $F_w = \frac{(b_1+n_2)t_w f_{yw}}{\gamma_{mo}} > V$

Where, $b_1$ = stiff bearing length = 0 to 100 mm

$n_2$ = 2.5 ( $t_f$ +$r_1$)

$f_{yw}$ = yield stress of web




**STEP 8: CHECK FOR DEFLECTION**

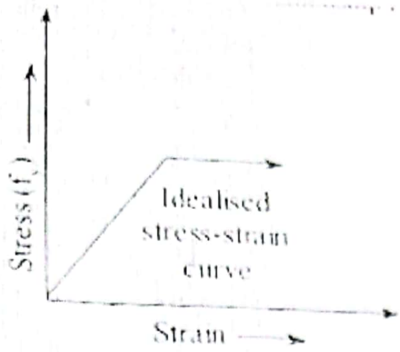
a. Actual deflection for simply supported

$$\delta_{max} = \frac{5}{384}\frac{wl^2}{EI}$$

b. Permissible deflection = Span/300 (table 6, Page no. 31, IS 800: 2007)

PLASTIC THEORY

**Assumptions made in plastic analysis of structure**




a. The strain hardening effect are ignored and the stress – strain relationship is expressed through two straight lines.

b. The effect of axial load on fully plastic moment capacity of the section are ignored.

c. The plane section before bending continuous to remain plane even after the bending. The shear deformation are ignored.

d. There is an identical relationship between compressive stress, compressive strain and tensile stress, tensile strain.

e. The effect of shear on a fully plastic moment capacity of the section is ignored.

f. A plastic hinge is formed at the cross section where the plastic moment is attained. This plastic hinge is allowed to undergo rotation of any magnitude, but at a fully plastic value, the bending moment remains uniform.

g. The materials is assumed to be homogeneous and isotropic in both the elastic and plastic states.

h. The resultant axial force on beam is zero

i. Total compression = total tension

j. The value of modulus of elasticity is same in both tension and compression.

k. The fibers in the lateral direction remains unaffected due to the expansion or contraction of longitudinal fibers

**Limitation of plastic analysis**

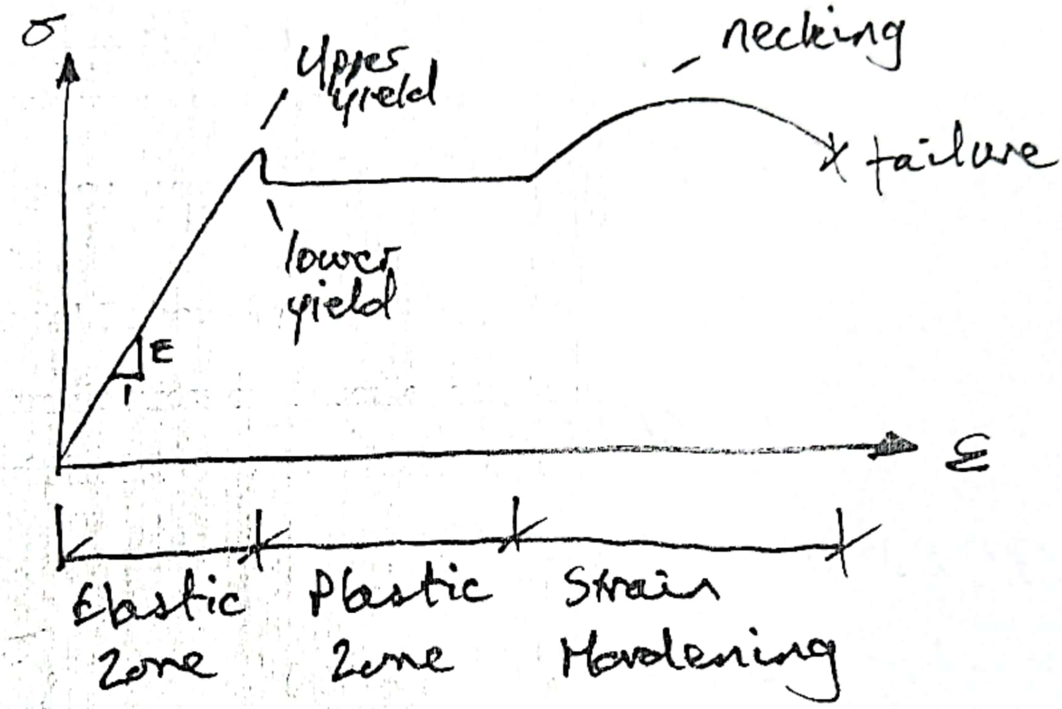
The limitation of the theory of plastic analysis are as follows

a. Only the material of ductile steel can be analyzed by plastic analysis. Therefore this method is not recommended for high strength steel.
b. It is difficult to recognize the unstable plastic structure than the identical elastic structure.
c. The connection provided in the plastic structure should be strong enough to transfer the plastic moments. Thus, plastic structure requires greater provision, care and good materials when compared to elastically designed structure.
d. Theory of simple plastic analysis is not applicable to the trusses in which the structural member carry axial forces instead of the bending. Hence, special methods are required to analysis such structure.

## 2. Development

### 2.1 Material Behaviour

A uniaxial tensile stress on a ductile material such as mild steel typically provides the following graph of stress versus strain:




As can be seen, the material can sustain strains far in excess of the strain at which yield occurs before failure. This property of the material is called its *ductility*.

Though complex models do exist to accurately reflect the above real behaviour of the material, the most common, and simplest, model is the *idealised stress-strain curve*. This is the curve for an ideal elastic-plastic material (which doesn't exist), and the graph is:

## Load Factor

$$\lambda = \frac{\text{collapse load}}{\text{Service load}} = \frac{P_c}{P}$$

## Shape factor ($\alpha$)

$$\alpha = \frac{M_p}{M_y} = \frac{Z_p}{Z_y}$$

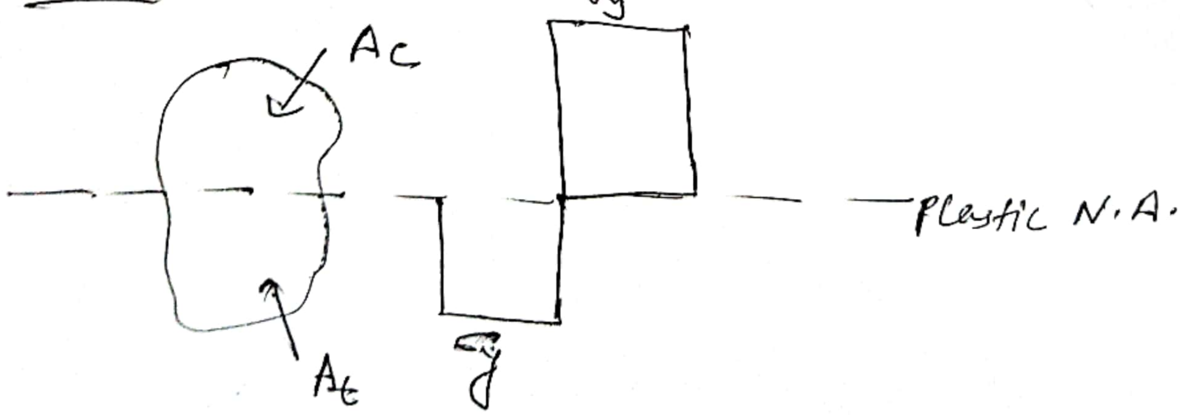
$M_p$ = Plastic moment capacity

$M_y$ = elastic moment capacity

$Z_p$ = plastic section modulus

$Z_y$ = Elastic section modulus.

## Plastic section modulus ($Z_p$)




$$Z_p = A_c \bar{y}_c + A_t \bar{y}_t$$

- Plastic Section Modulus

$$\boxed{Z_p = A_c\bar{y}_c + A_t\bar{y}_t}$$

**Remember**

Total compressive force = Total tensile force

Note that in plastic analysis the Neutral axis divides the cross-section into two equal halves whereas in elastic analysis NA. Passes through C.G.

i.e., $A_c = A_t = A/2$, where A is total area

## SHAPE FACTOR (α)

$$\alpha = \frac{M_p}{M_y} = \frac{Z_p}{Z_y}$$ Where $Z_y$ is elastic section modulus

$$\text{Load factors } (\lambda) = \frac{P_c}{P}$$

Load Factor = Factor of safety × Shape factor  $\lambda = FS \times \alpha$

## SHAPE FACTORS FOR DIFFERENT SHAPES

| Section | Shape Factor (α) |
|---|---|
| 1. Rectangular section | 1.5 |
| 2. (a) Triangular section (vertex upward) | 2.34 |
| (b) Triangular section (vertex horizontal) | 2.00 |
| 3. Solid circular section | 1.7 |
| 4. Hollow circular section | $1.7 \times \frac{(1-K^3)}{(1-K^4)}$ |
| 5. Thin circular ring solid | 1.27 |
| 6. (a) Diamond section (Rhombus) | 2.00 |
| (b) Thin hollow rhombus | 1.50 |
| 7. I-section | |
| (a) About strong axis | ≈ 1.12 |
| (b) About weak axis | ≈ 1.55 |
| 8. T-section | ≈ 1.90 to 1.95 |

Where K = Ratio of inner diameter to outer diameter.

## COLLAPSE LOADS

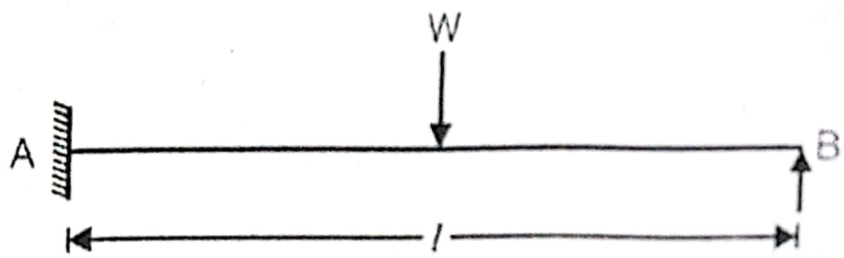
1. Simply supported beam with concentrated load at the center

$$W_c = \frac{4M_p}{l}$$

2. Simply supported beam with uniformly distributed load

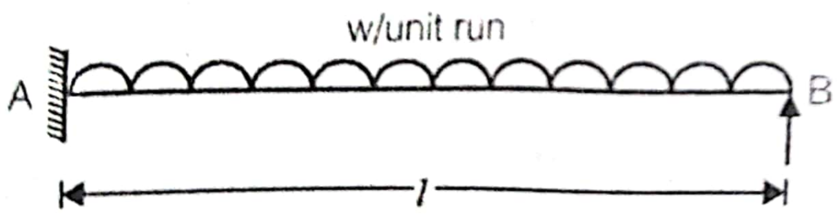
$$W_c = \frac{16M_p}{l}$$

3. Propped cantilever with concentrated load at the center



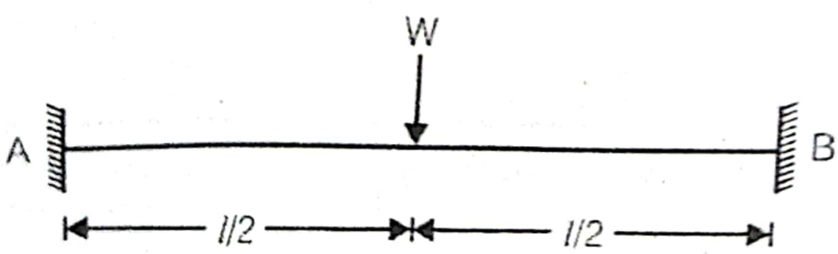

$$W_c = \frac{6M_p}{l}$$

4. Propped cantilever with uniformly distributed load



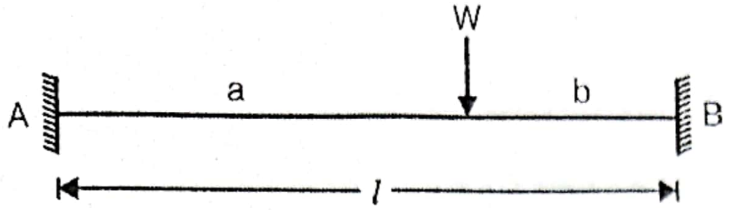

$$W_c = \frac{11.656M_p}{l}$$

5. Fixed beam with concentrated load at the center



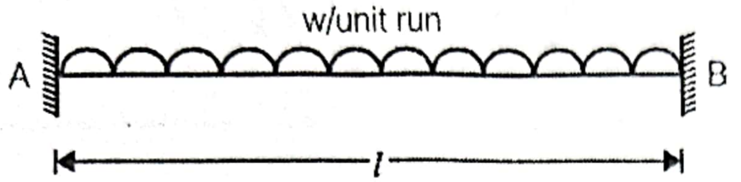

$$W_c = \frac{8M_p}{l}$$

6. Fixed beam with eccentric loading



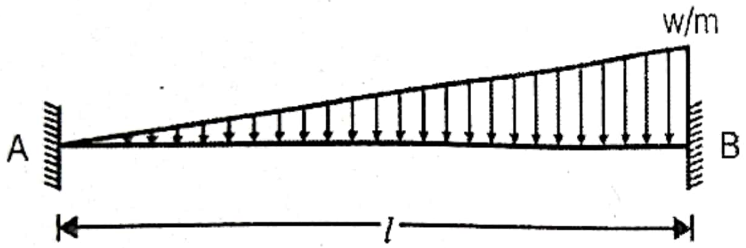

$$W_c = \frac{2l}{ab} M_p$$

7. Fixed beam with uniformly distributed load



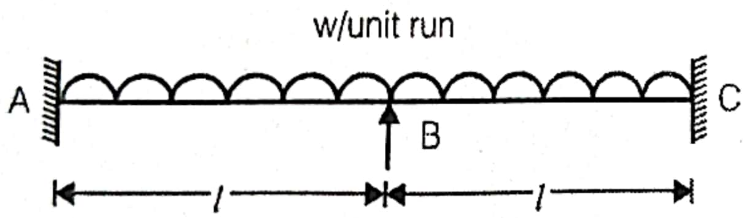

$$w_c = \frac{16M_p}{l^2}$$

8. Fixed beam with hydrostatic loading




$$w_c = \frac{18\sqrt{3}\,M_p}{l^2}$$

9. Continuous beam with uniformly distributed load




$$w_c = \frac{11.656\,M_p}{l^2}$$

The positions of the plastic hinges are one at the support B and one on each side of the central support at a distance of 0.414 $l$ from A & C.

**Remember**

- **Upper bound theorem**

  It satisfies equilibrium and mechanism condition. $P \geq P_u$

- **Lower bound theorem**

  It satisfies equilibrium and yield condition. $P \leq P_u$

## Factor of Safety

This is defined as:

$$\text{FoS} = \frac{\text{First yield load}}{\text{Working Load}}$$

$$\text{Shape Factor, SF} = \frac{\text{Plastic Moment}}{\text{Yield Moment}}$$

Shape factor of every type cross-section is always greater than 1 i.e.

**SF > 1**

**Plastic Moment > Yield Moment**

...restraints are placed at the limiting distance $L_m$ ... is required

## Collapse load of standard cases

### *Concentrated Load at Centre*

**Simply Supported Beam**

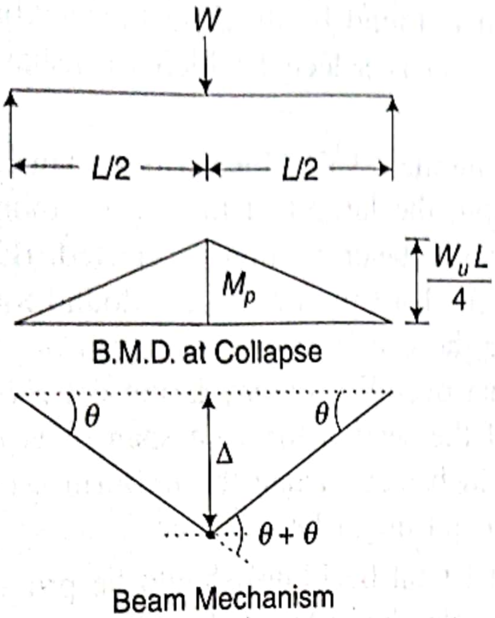



**Fig. 3.17**

*Static method*

$$\frac{W_u L}{4} = M_p$$

or $$W_u = \frac{4M_p}{L}$$

*Kinematic method*

$$W_u\left(\frac{L}{2}\right)\theta = M_p(\theta + \theta)$$

or $$W_u = \frac{4M_p}{L}$$

**Fixed Beam**

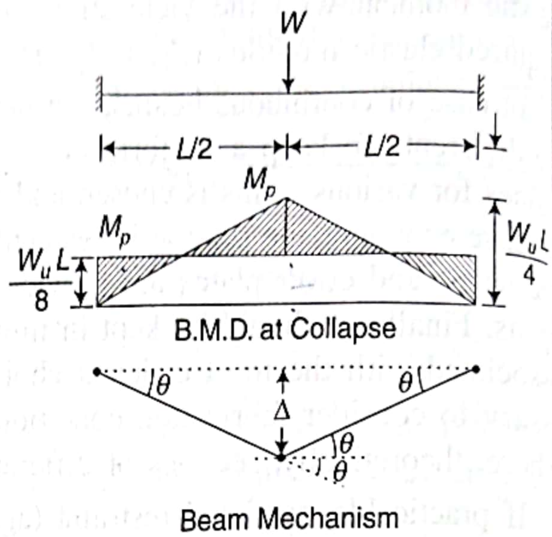



**Fig. 3.18**

*Static method*

$$\frac{W_u L}{4} = 2M_p$$

or $$W_u = \frac{8M_p}{L}$$

*Kinematic method*

$$W_u\left(\frac{L}{2}\right)\theta = M_p\theta + M_p(\theta + \theta) + M_p\theta$$

or $$W_u = \frac{8M_p}{L}$$

### *Eccentric Load*

**Simply Supported Beam**

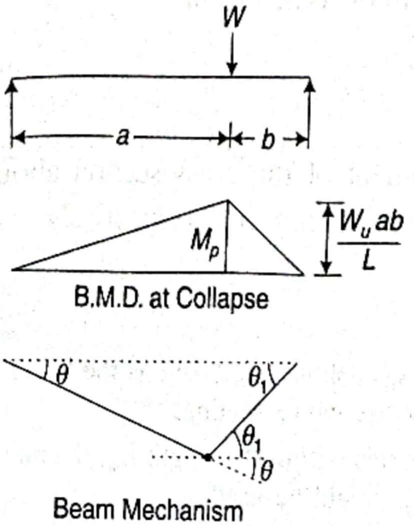



**Fig. 3.19**

**Fixed Beam**

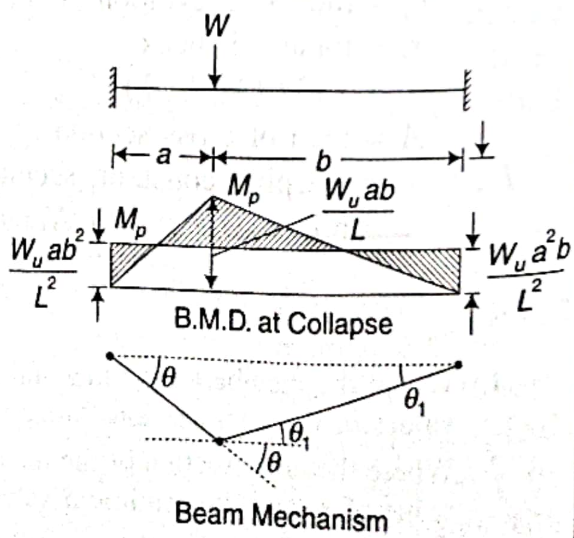



**Fig. 3.20**

*Static method*

$$\frac{W_u ab}{L} = M_p$$

or $$W_u = \frac{M_p L}{ab}$$

*Kinematic method*

$$W_u a\theta = M_p(\theta + \theta_1)$$

$$= M_p\left(\theta + \frac{a}{b}\theta\right)$$

or $$W_u a\theta = M_p\,\theta\left(1 + \frac{a}{b}\right)$$

$$= M_p\,\theta\left(\frac{b+a}{b}\right)$$

$$W_u a\theta = M_p\,\theta\left(\frac{L}{b}\right)$$

or $$W_u = M_p\frac{L}{ab}$$

*Static method*

$$\frac{W_u ab}{L} = 2M_p$$

or $$W_u = \frac{2M_p L}{ab}$$

*Kinematic method*

$$W_u a\theta = M_p\theta + M_p(\theta + \theta_1) + M_p\theta_1$$

or $$W_u a\theta = M_p\theta + M_p\left(\theta + \frac{a}{b}\theta\right) + M_p\frac{a}{b}\theta$$

$$= 2M_p\theta\left(\frac{a+b}{b}\right)$$

$$= 2M_p\,\theta\frac{L}{b}$$

or $$W_u = \frac{2L}{ab}M_p$$

or $$W_u = \frac{2M_p L}{ab}$$

*Uniformly Load Distributed*

| *Simply Supported Beam* | *Fixed Beam* |
|---|---|

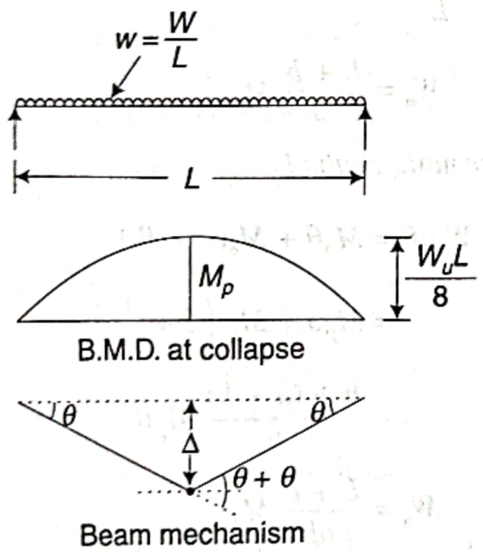



**Fig. 3.21**

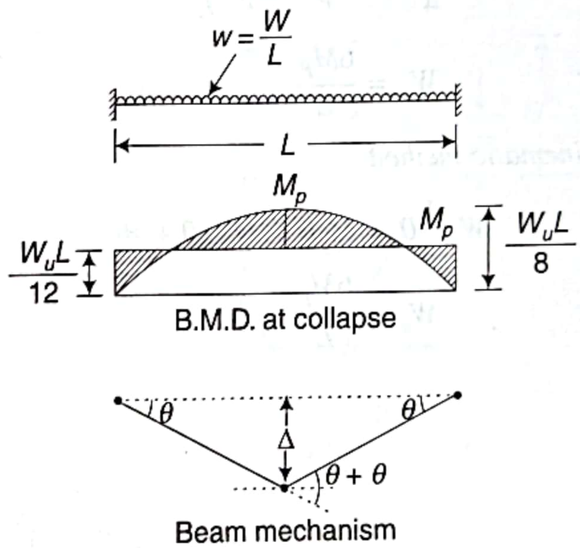



**Fig. 3.22**

*Static method*

$$\frac{W_u L}{8} = M_p$$

$$W_u = \frac{8M_p}{L}$$

*Kinematic method*

$$W_u\frac{1}{2}\left(\frac{L}{2}\theta\right) = M_p(\theta + \theta)$$

or $$W_u = \frac{8M_p}{L}$$

*Static method*

$$\frac{W_u L}{8} = 2M_p$$

$$W_u = \frac{16M_p}{L}$$

*Kinematic method*

$$W_u\frac{1}{2}\left(\frac{L}{2}\theta\right) = M_p\theta + M_p(\theta + \theta) + M_p\theta$$

or $$W_u = \frac{16M_P}{L}$$

**Concentrated Load**

*Propped Cantilever*

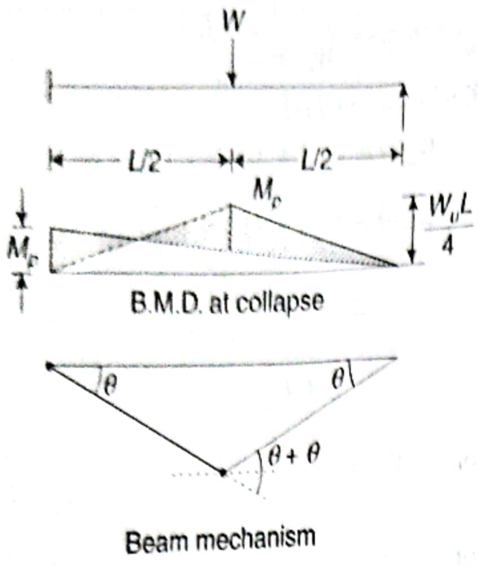



Fig. 3.23

*Static method*

$$\frac{W_u L}{4} = M_p + M_p \frac{L/2}{L}$$

or $$W_u = \frac{6M_p}{L}$$

*Kinematic method*

$$W_u \frac{L}{2}\theta = M_p\theta + M_p(\theta + \theta)$$

or $$W_u = \frac{6M_p}{L}$$

*Propped Cantilever*

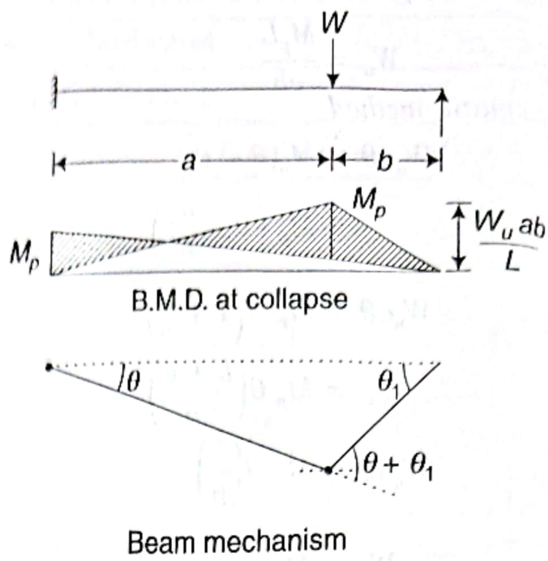



Fig. 3.24

*Static method*

$$\frac{W_u ab}{L} =$$

or $$W_u = \frac{L+b}{ab} M_p$$

*Kinematic method*

$$W_u a\theta = M_p\theta + M_p(\theta + \theta_1)$$

or $$= M_p\theta + M_p\left(\theta + \frac{a}{b}\theta\right)$$

or $$= \frac{b + (a + b)}{b} M_p\theta$$

or $$W_u = \frac{L+b}{ab} M_p$$

## Solved Examples

***Example 3.1*** Find the shape factors for the following sections:

(a) Square of side $a$ with its diagonal parallel to the $zz$-axis.

(b) A diamond section with unequal diagonals, the shorter being $b$ and longer $h$; the shorter diagonal placed parallel to the $zz$-axis.

(c) Hollow tube section of external diameter $D$ and internal diameter $d$.

(d) Triangular section of base $b$ and height $h$.

(e) Tee-section as shown in Fig. Ex. 3.1 (e).

## *Solution*

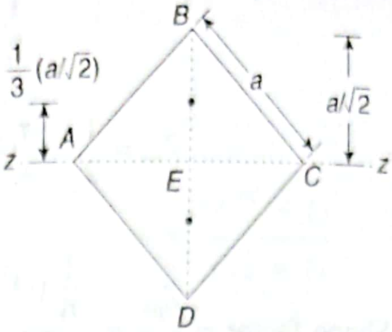
(a) Refer Fig. Ex. 3.1(a).

$$AC = \sqrt{a^2 + a^2} = \sqrt{2}a$$

$$BE = \frac{BD}{2} = \frac{AC}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$$

Moment of inertia about $zz$-axis,

$$I_z = \frac{2\sqrt{2}}{12}\, a \left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{12}$$




(a)
**Fig. Ex. 3.1**

Elastic section modulus,

$$Z_{ez} = \frac{a^4/12}{a/\sqrt{2}} = \frac{a^3}{6\sqrt{2}}$$

Plastic section modulus,

$$Z_{pz} = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

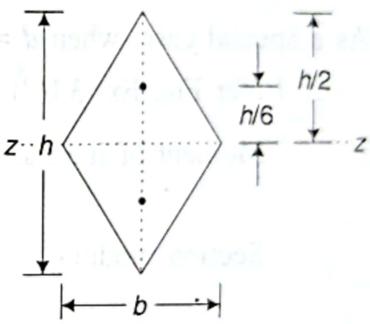
$$= \frac{a^2}{2} \times \left(\frac{a}{3\sqrt{2}} + \frac{a}{3\sqrt{2}}\right) = \frac{a^3}{3\sqrt{2}}$$

$$\text{Shape factor } S = \frac{Z_{pz}}{Z_{ez}} = \frac{a^3/3\sqrt{2}}{a^3/6\sqrt{2}} = 2$$

(b) (Ref. Fig. Ex. 3.1(b))

Moment of inertia about $zz$-axis,

$$I_z = 2 \times \frac{1}{12}\, b \left(\frac{h}{2}\right)^3 = \frac{bh^3}{48}$$




(b)
**Fig. Ex. 3.1**

Elastic section modulus,

$$Z_{ez} = \frac{bh^3/48}{h/2} = \frac{bh^2}{24}$$

Plastic section modulus,

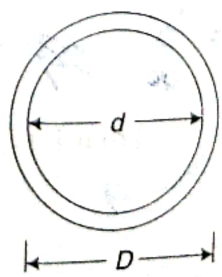
$$Z_{pz} = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{1}{2} \times \left(\frac{bh}{2}\right) \times \left(\frac{h}{6} + \frac{h}{6}\right) = \frac{bh^2}{12}$$

$$\text{Shape factor } S = \frac{bh^2/12}{bh^2/24} = 2$$

(c) Refer Fig. Ex. 3.1(c)

Moment of inertia about $zz$-axis,

$$I_z = \frac{\pi}{64}(D^4 - d^4)$$




(c)
**Fig. Ex. 3.1**

Elastic section modulus,

$$Z_{ez} = \frac{\pi}{64} \times \frac{(D^4 - d^4)}{D/2} = \frac{\pi}{32} \times \frac{D^4 - d^4}{D}$$

Plastic section modulus, $Z_{pz} = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$\bar{y}_1 = \bar{y}_2 = \frac{\frac{1}{2} \times \left[\frac{\pi D^2}{4}\left(\frac{2}{3} \times \frac{D}{\pi}\right) - \frac{\pi d^2}{4}\left(\frac{2}{3} \times \frac{d}{\pi}\right)\right]}{\frac{1}{2} \times \left[\frac{\pi}{4}(D^2 - d^2)\right]} = \frac{2}{3\pi} \times \frac{(D^3 - d^3)}{(D^2 - d^2)}$$

$$Z_{pz} = \frac{1}{2} \times \frac{\pi}{4}(D^2 - d^2)\left[2 \times \frac{2}{3\pi} \times \frac{(D^3 - d^3)}{(D^2 - d^2)}\right] = \frac{1}{6}(D^3 - d^3)$$

$$\text{Shape factor} = \frac{Z_{pz}}{Z_{ez}} = \frac{\frac{1}{6}(D^3 - d^3)}{\frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)} = \frac{32}{6\pi} \times \frac{D^3\left(1 - \frac{d^3}{D^3}\right)}{D^3\left(1 - \frac{d^4}{D^4}\right)} = \frac{32}{6\pi} \times \frac{\left(1 - \frac{d^3}{D^3}\right)}{\left(1 - \frac{d^4}{D^4}\right)}$$

Let $$k = \frac{d}{D}$$

$$\text{Shape factor} = \frac{32}{6\pi} \times \frac{(1 - k^3)}{(1 - k^4)} = 1.7\frac{(1 - k^3)}{(1 - k^4)}$$

when $D \approx d,\quad k = 3/4$

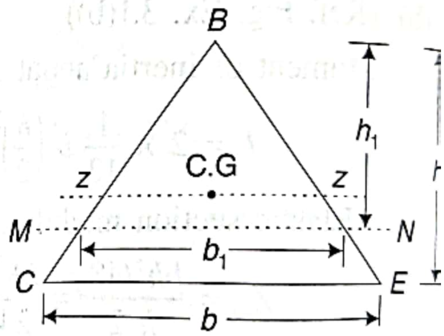
Shape factor = 1.27

As a special case, when $d = 0$, i.e., for a circular plate, $k = 1$ shape factor = 1.

(d) Refer Fig. Ex. 3.1(d).

Moment of inertia, $$I_z = \frac{bh^3}{36}$$

Section modulus, $$Z_{ez} = \frac{bh^3/36}{(2/3)h} = \frac{bh^2}{24}$$




(d)

**Fig. Ex. 3.1**

Plastic section modulus, $$Z_{pz} = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$$

Let $MN$ be the equal area axis; then

$$\frac{1}{2} b_1 h_1 = \frac{1}{2} \times \left(\frac{bh}{2}\right) \Rightarrow b_1 h_1 = \left(\frac{bh}{2}\right) \quad (1)$$

$$\frac{h_1}{b_1} = \frac{h}{b} \quad (2) \quad \text{(From similar } \Delta^s\ BMN \text{ and } BCE)$$

From Eqs. (1) and (2),

$$h_1 = \frac{h}{\sqrt{2}}$$

and $$b_1 = b/\sqrt{2}$$

$$A = \frac{bh}{2}$$

$$\bar{y}_1 = \frac{h_1}{3} = \frac{h}{3\sqrt{2}}$$

$$\bar{y}_2 = \left(\frac{h - h_1}{3}\right) \times \left(\frac{b_1 + 2b}{b_1 + b}\right)$$

$$= \left(\frac{h - \frac{h}{\sqrt{2}}}{3}\right) \times \left(\frac{2b + \frac{b}{\sqrt{2}}}{b + \frac{b}{\sqrt{2}}}\right) = \frac{h(\sqrt{2} - 1)}{3\sqrt{2}} \times \frac{(2\sqrt{2} + 1)}{(\sqrt{2} + 1)}$$

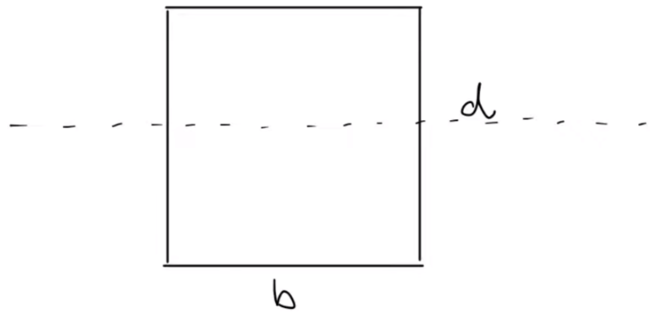
$$= \frac{0.4643h}{3}$$

$$Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2) = \frac{1}{2} \times \frac{bh}{2} \times \left[\frac{h}{3} \times \frac{1}{\sqrt{2}} + \frac{h}{3} \times 0.4643\right]$$

$$= \frac{bh^2}{12} \times [0.7072 + 0.4643] = 0.0976\ bh^2$$

$$\text{Shape factor } S = \frac{Z_{pz}}{Z_{ez}} = \frac{0.0976\ bh^2}{bh^2/24} = 2.343$$

(i) Rectangular section.



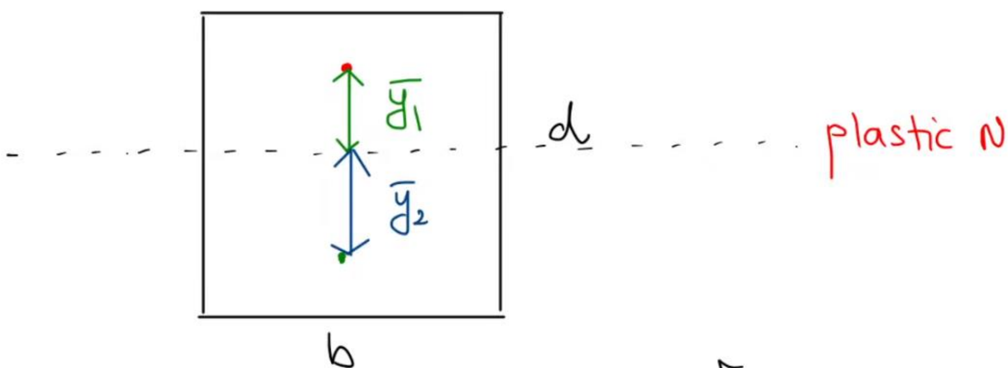

$$Z = \frac{I}{y}$$

$$= \frac{bd^3/12}{d/2}$$

$$Z = \frac{bd^2}{6}$$

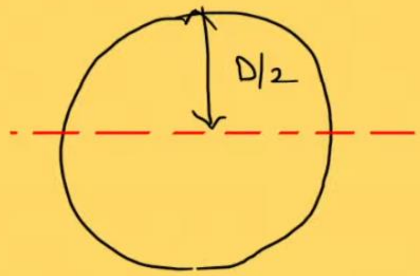
(i) Rectangular section.

$$\bar{y}_1 = \frac{d}{4} = $$




$$Z_p = \frac{A}{2}\left[\bar{y}_1 + \bar{y}_2\right] = \frac{bd}{2}\left[\frac{d}{4} + \frac{d}{4}\right.$$

## (ii) Circular section:

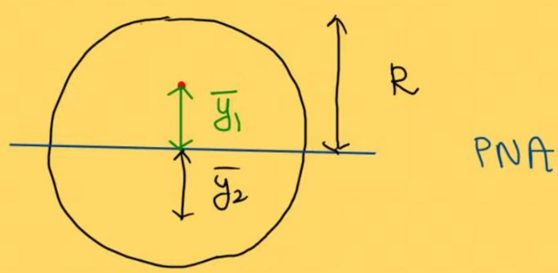



$$Z = \frac{I}{y} = \frac{\frac{\pi}{64} D^{4}}{D/2} = \frac{\pi D^3}{32}$$

## (ii) Circular section:

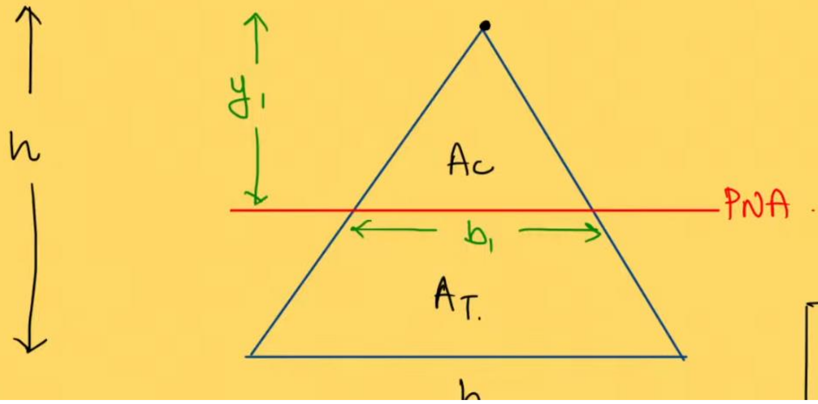
$Z = \pi D^3 / 32$

$\bar{y}_1 = \frac{4}{3} \frac{R}{\pi}$




$$Z_p = \frac{A}{2}\left[\bar{y}_1 + \bar{y}_2\right] = \frac{\frac{\pi}{4} D^2}{2}\left[\frac{4}{3}\frac{D/2}{\pi} + \frac{4}{3}\frac{D/2}{\pi}\right]$$

# Plastic Analysis




$$A = \frac{1}{2} bh$$

$$A_c = A_T = \frac{A}{2}$$

$$y_1 = ?$$

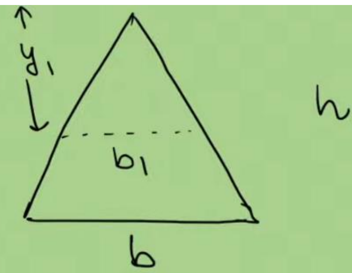
$$A_c = \frac{1}{2} b_1 y_1$$

$$\frac{A}{2} = \frac{1}{2} b_1 y_1$$

$$\frac{\frac{1}{2} bh}{2} = \frac{1}{2} \left(b_1 y_1\right)$$

$$y_1 b_1 = \frac{bh}{2} \quad \text{———} \quad (1)$$

Similar Δ :




$$\frac{y_1}{h} = \frac{b_1}{b}$$

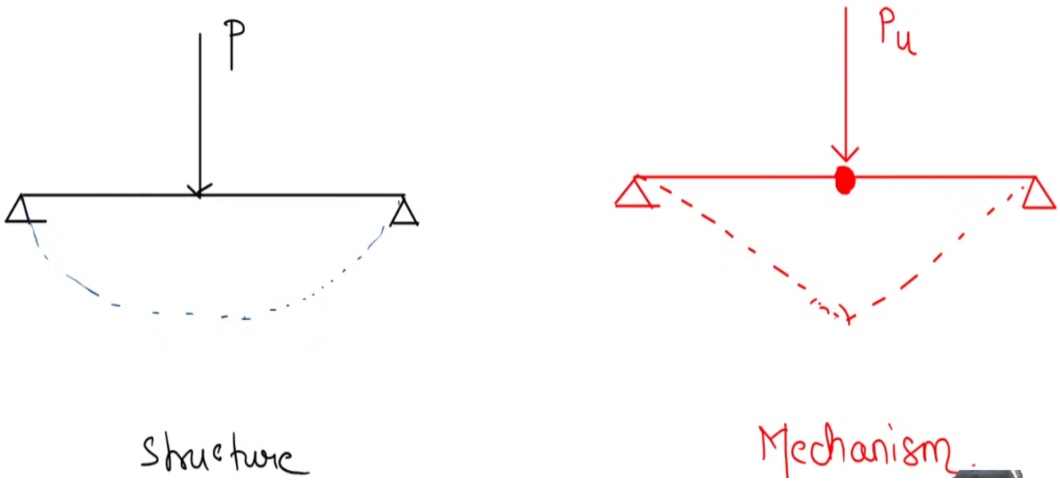
$$b_1 = \frac{b}{h} \cdot y_1$$

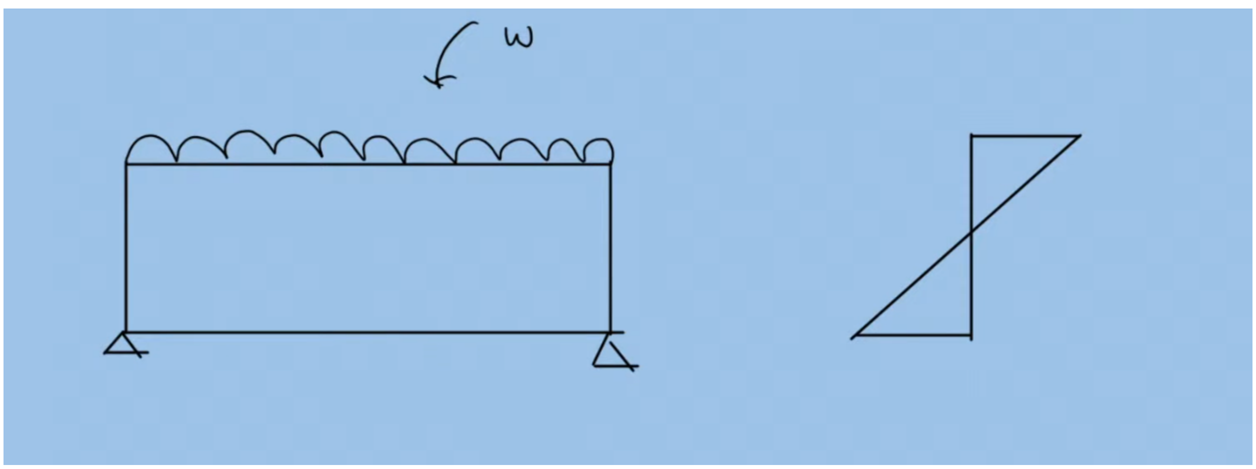
$$y_1 = \frac{bh}{2b_1} = \frac{bh}{2 \cdot \left( \frac{b}{h} \cdot y_1 \right)}$$

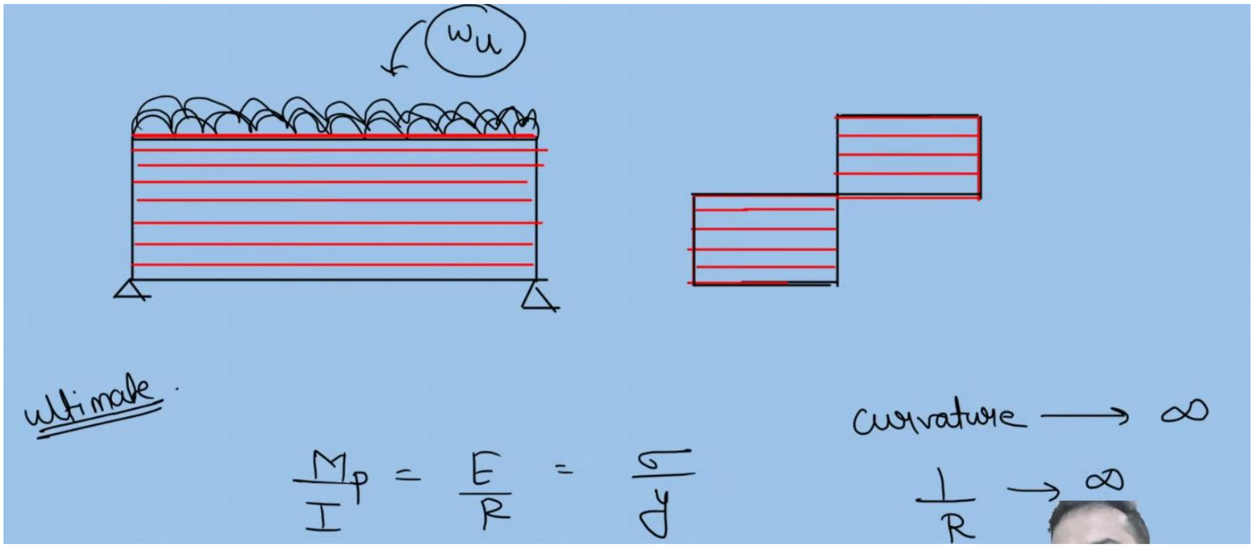
$$y_1^2 = \frac{h^2}{2}$$

$$\boxed{y_1 = \frac{h}{\sqrt{2}}}$$

* How to find the collapse load :

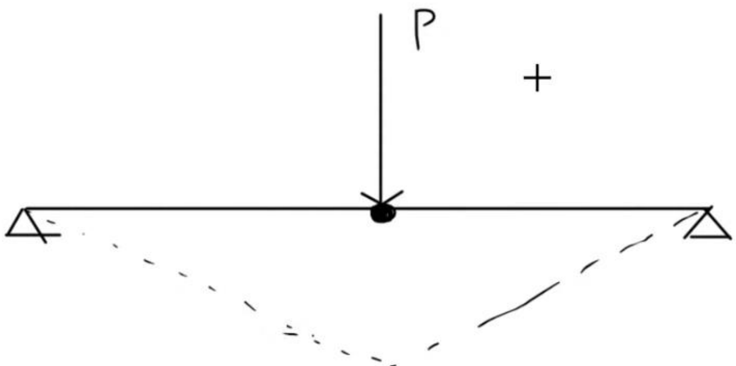




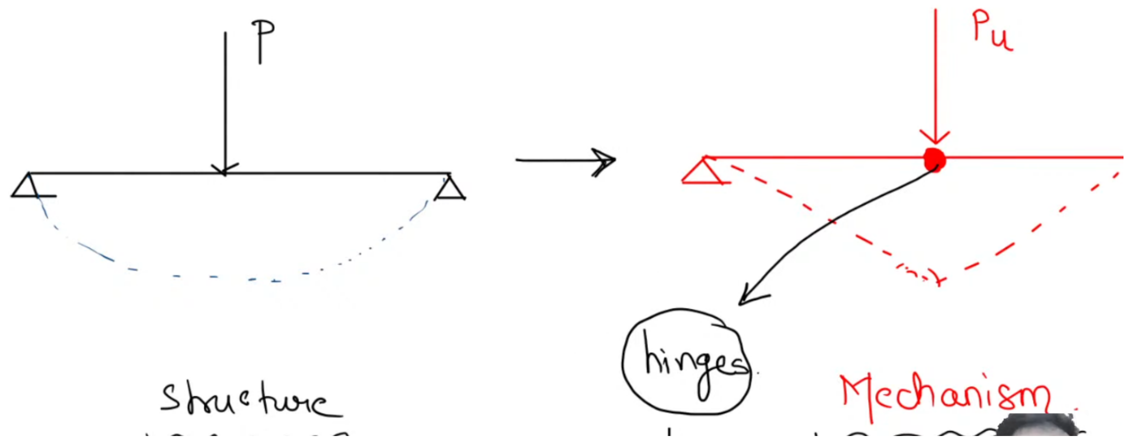

R=0

Plastic hinge ⟶ curvature = ∞

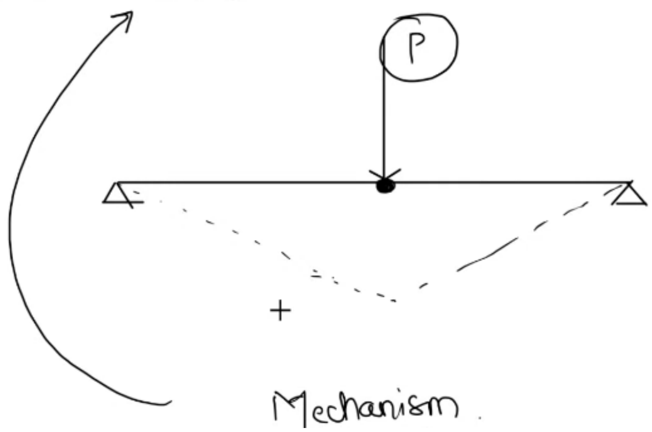



Mechanism.

## ★ How to find the collapse load :




Plastic hinge → curvature = ∞.




$M \rightarrow M_p$

Collapse load → $w_u$, $P_u$ ?

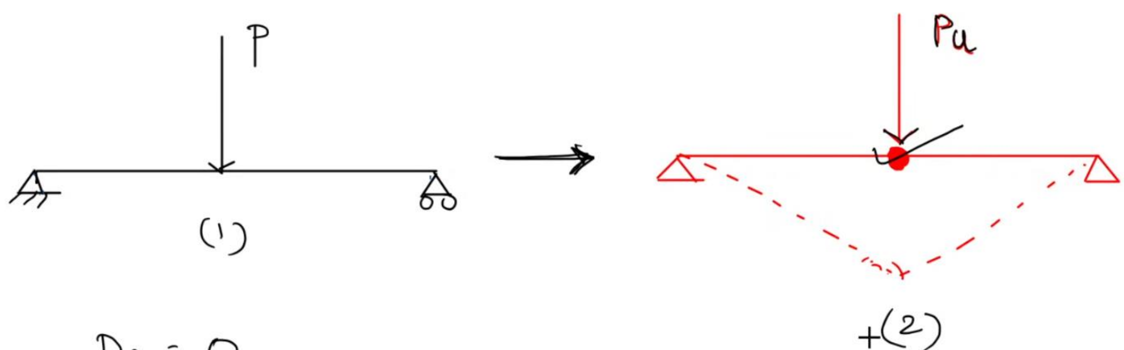
(i) Structure → mechanism (+ with the help of hinges).

(ii) How to calculate no. of plastic hinge.

$$n = D_s + 1$$

Structure ⟶ mechanism.

★ <u>How to find the collapse load</u>:




$D_s = 0$.

$n = 1$

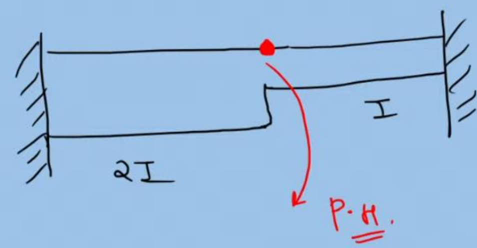
(iii) <u>Position of plastic hinge.</u>

→ location of max. BM.

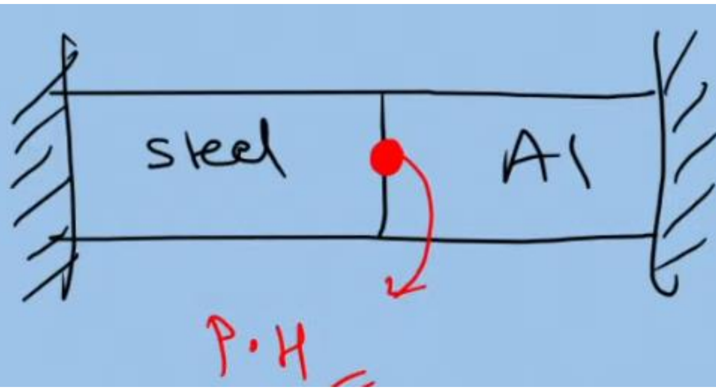
→ fixed support, rigid joint.

→ Under point load in SS beam.

→ at the location where c/s changes.

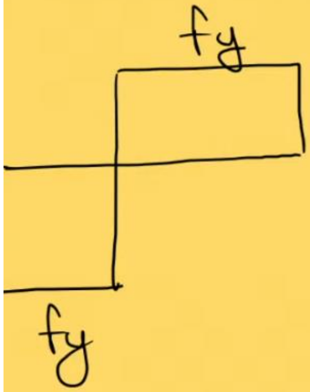



→ at the location where material changes.



★ Calculation of collapse load :

Principle of virtual work done.




→ Equilibrium condition

→ Mechanism condition.

→ Yield condition.

★ Calculation of collapse load :

Principle of virtual work done.

→ Equilibrium condition ✓ $\Sigma f_x = 0$, $\Sigma f_y = 0$, $\Sigma M_z = 0$

→ Mechanism condition ✓

→ Yield condition ✓

## Principle of V·W·D

Total virtual work done = 0.

External WD + Internal WD = 0

External WD → load

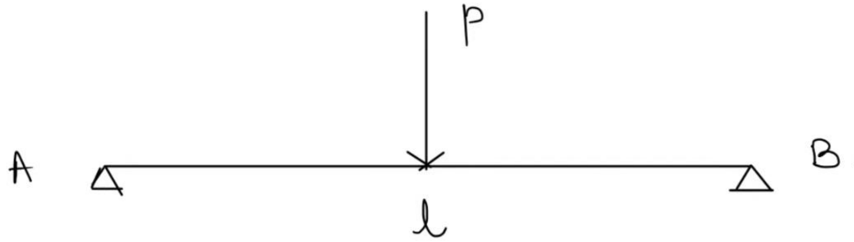
Internal WD → $M_p$

External WD = load × deflection.

Internal WD = Moment × rotation. (−ve)

opposite.

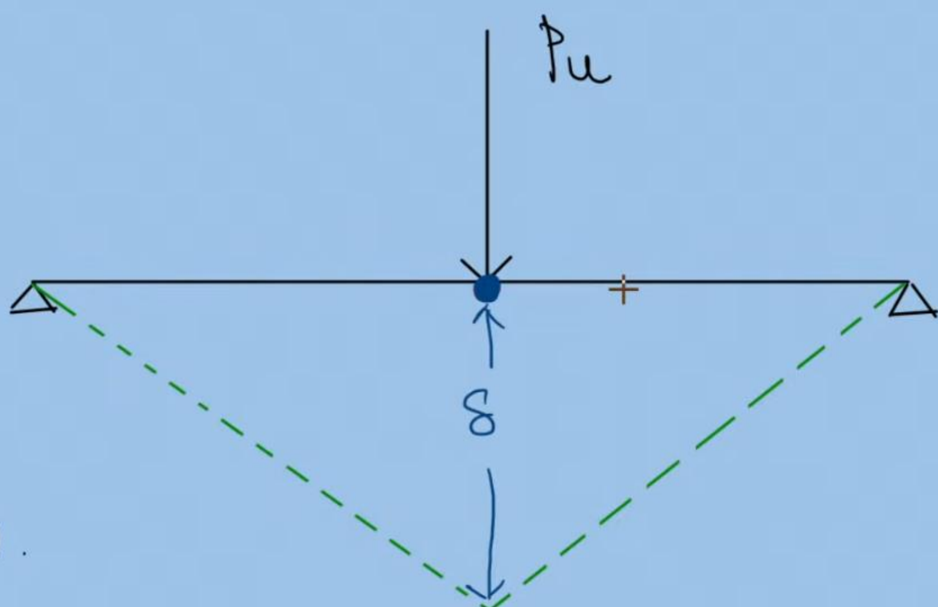
Qu. Find the collapse load for SS beam subjected to concentrated load at centre.




Step: 1 Structure ⟶ Mechanism

(i) $D_S = 0$.

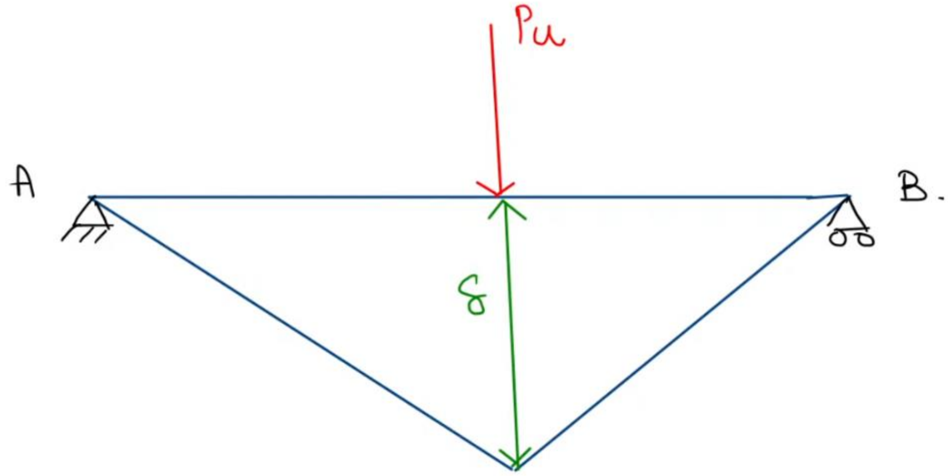
(ii) $n = D_S + 1 = 1$

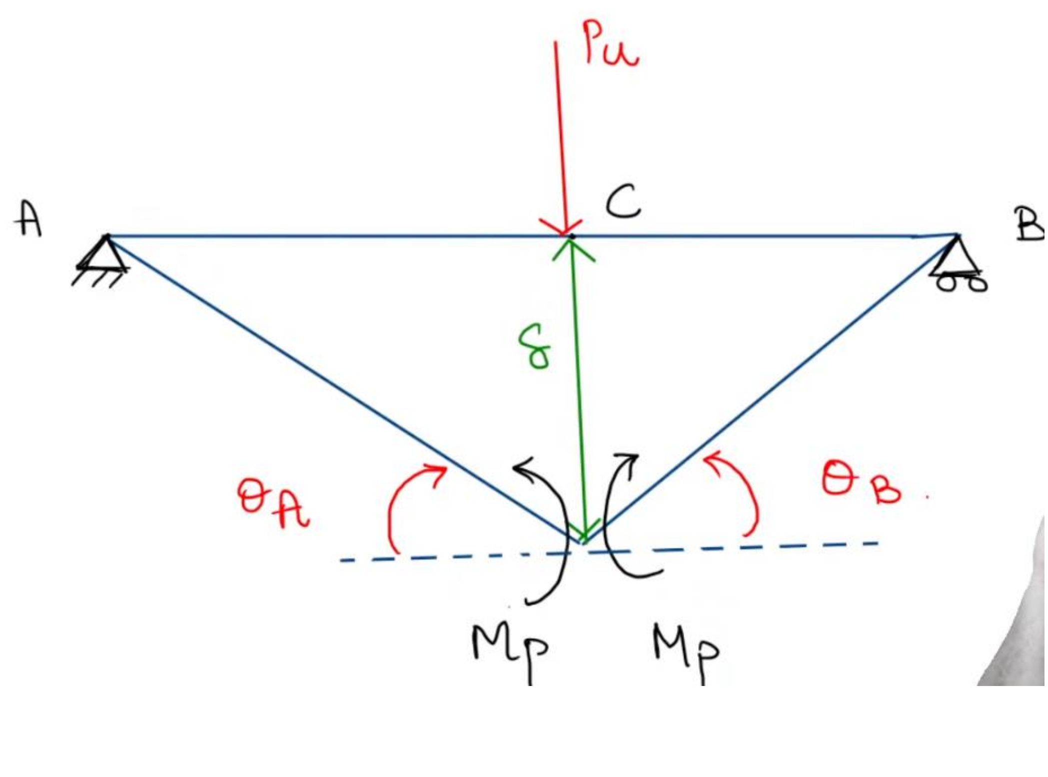



Mechanism.

(iii) Principle of virtual work done:

External WD = Internal WD.



Internal WD = $M_p\theta + M_p\theta$

Internal WD = $2M_p\theta$.

★ Principle of V.W.D ⇒ External = Internal

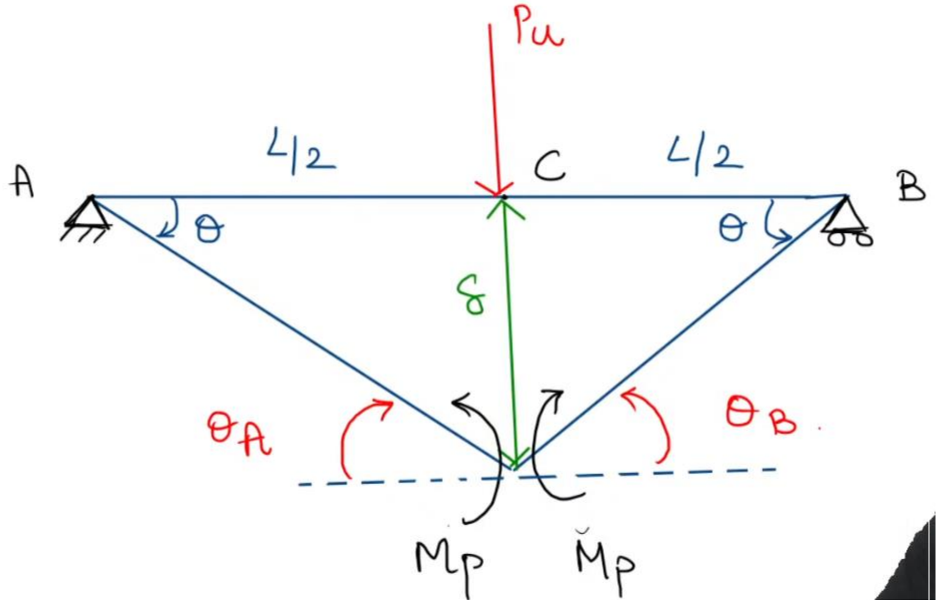
$$P_u \cdot \delta = 2M_p\theta.$$

★ Principle of V.W.D ⇒ External = Internal

$$P_u \cdot \delta = 2M_p\theta.$$

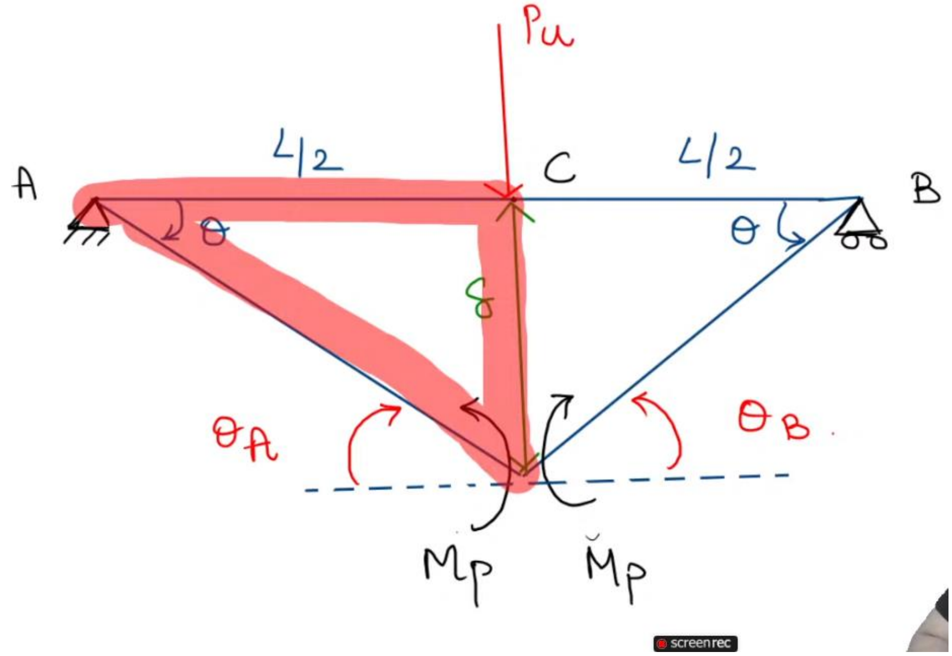
(iii) <u>Principle of virtual work done :</u>

External WD = $P_u \cdot \delta$ .



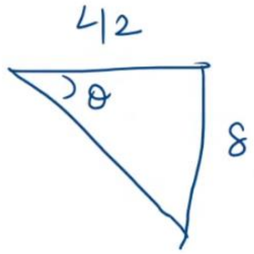
(iii) <u>Principle of virtual work done</u> :

External WD = $P_u \cdot \delta$ .

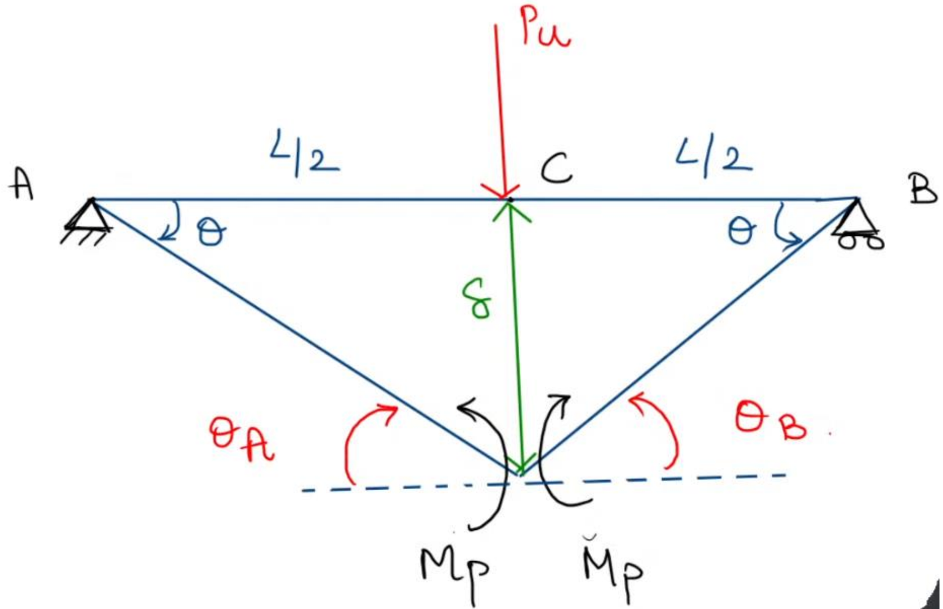


(iii) **Principle of virtual work done :**

External WD = $P_u \cdot \delta$




$$\delta = \theta \cdot \frac{L}{2}$$




$$P_u \cdot \delta = 2 M_p \theta$$

$$P_u \cdot \left(\theta \times \frac{L}{2}\right) = +2 M_p \theta$$

$$P_u = \frac{4 M_p}{L}$$

+

# What is a Column Base?

The column base transmits the column load to the concrete or masonry foundation blocks. In order to keep the intensity of bearing pressure on the foundation block within the bearing strength, the column base distributes the load over a larger surface.

### Column Base Definiton

The column base is the interface between the columns and their foundation. Column base reduces the intensity of loading and distributing it over the foundation. The area of the column base or base plate is chosen so that the intensity of load distributed is less than the bearing capacity of concrete on which the plate resists. It also maintains the proper alignment of columns in a plan.

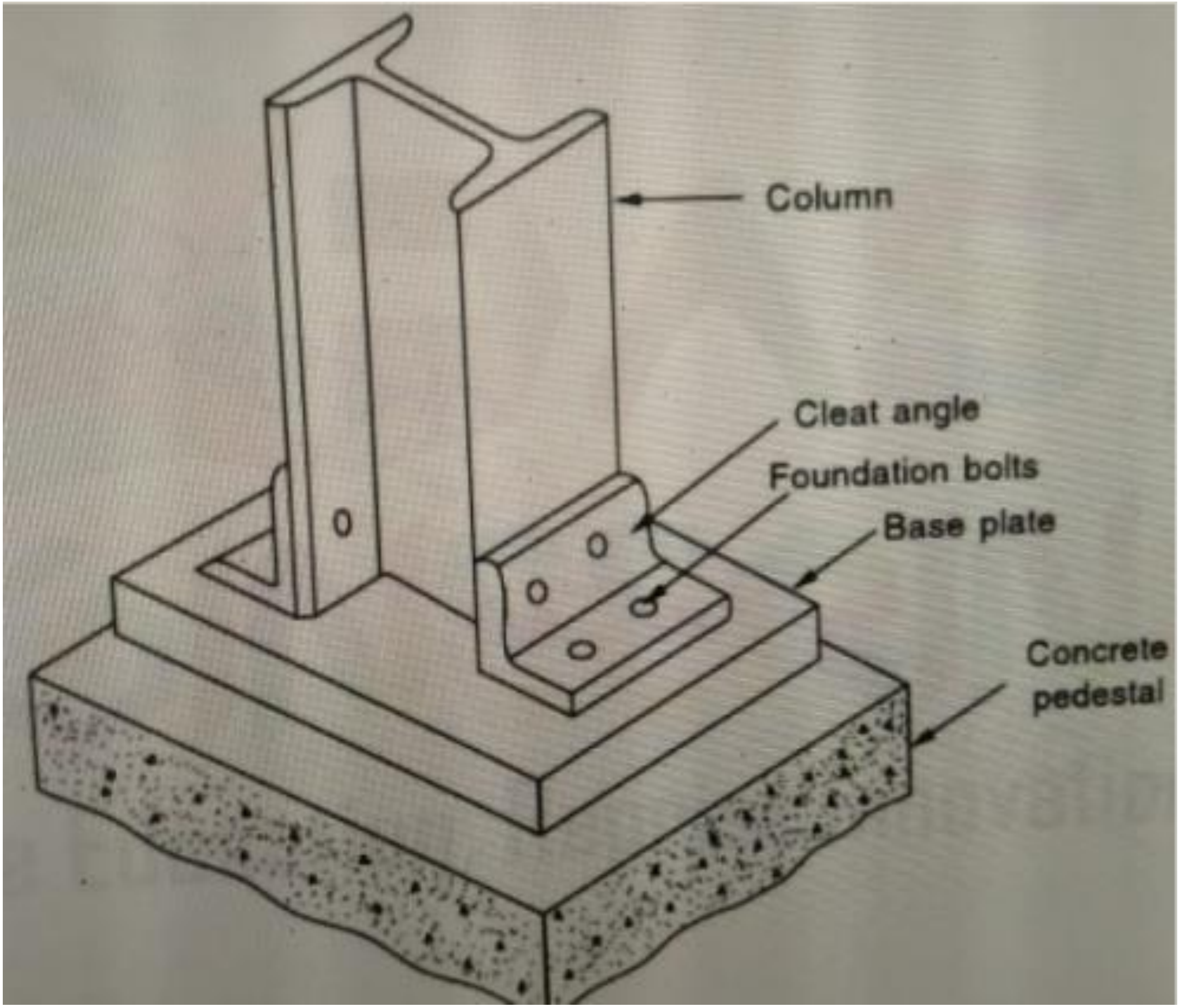
# Types of Column Base

Based on the stability of the foundation and on the bases in the steel column, column bases are classified on the basis of transmitting direct load and secondary carrying an appreciable bending moment in addition to the direct load. Column base types are:-

- Slab base
- Gusset base
- Grillage foundation

# Slab Base

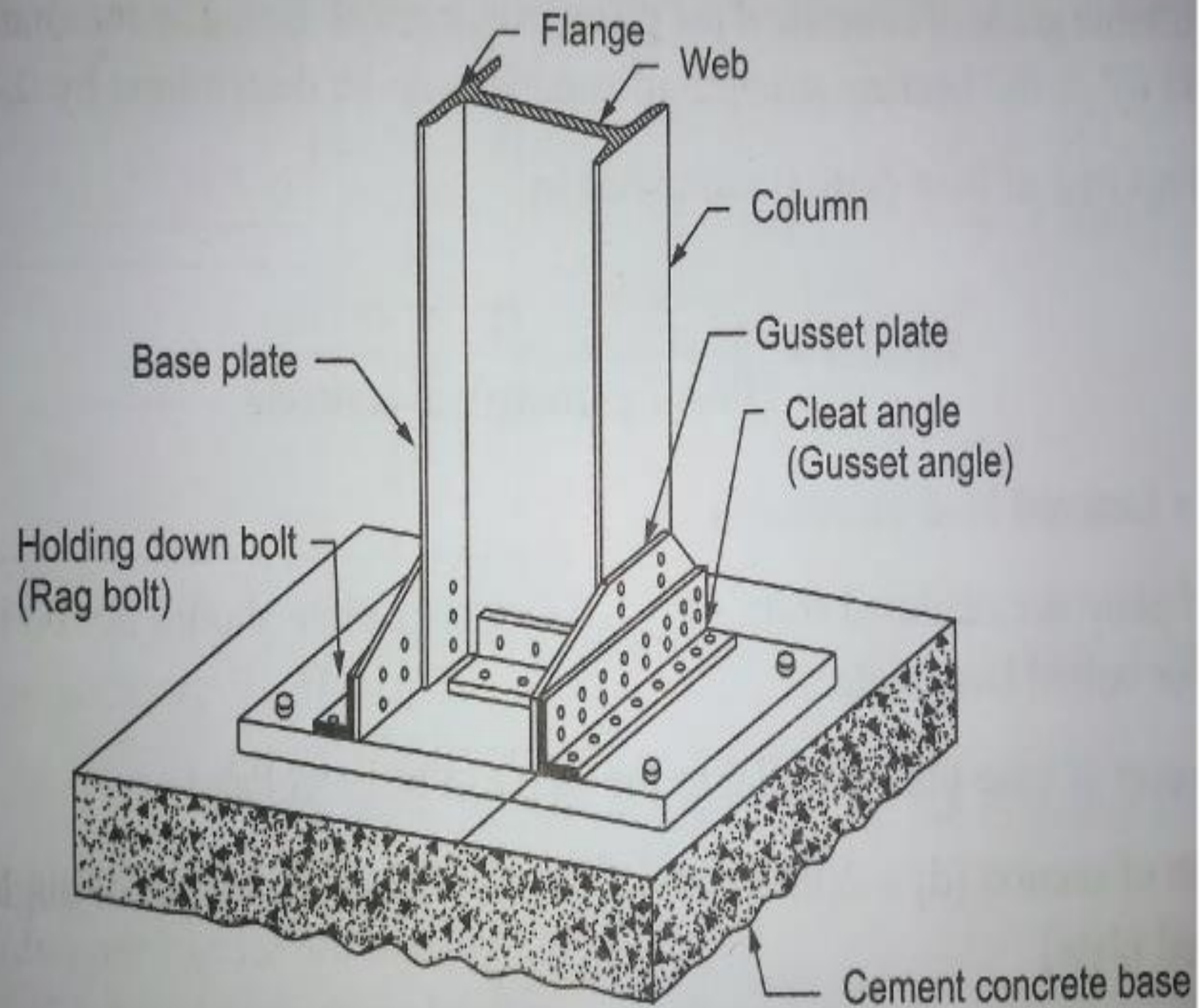
Slab base is provided to the column subjected to only axial load. A steel plate is used to transfer the load to the concrete pedestal in the case of a slab base. In the slab base, the column is connected to a cleat angle. The critical moment will be at the edge of the column.




## Gusset Base

A gusseted base is provided when the heavy load on the column or axial load is accompanied by a bending moment. The column is connected to the base plate through the gusset angle and gusset plate. In this type of base, the load is transferred to the bearing and gusset simultaneously.

Gusset material ( two gusset plates and two gusset angles) is used in the base to increase the bearing area consequently it reduces the thickness of the base as compared to the slab base. The diagram shows how they are connected with each other.

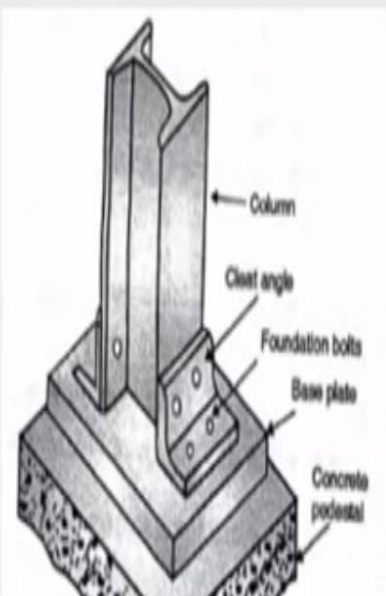
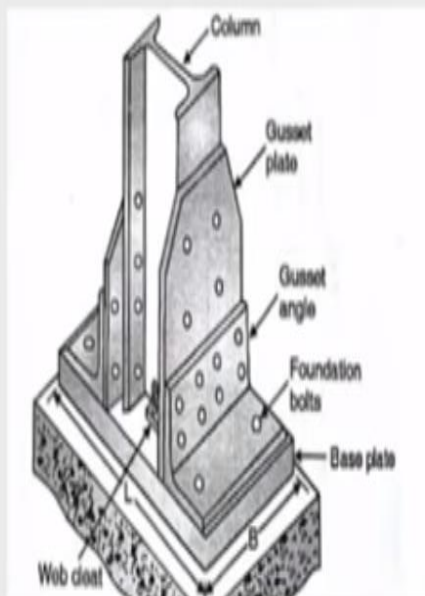


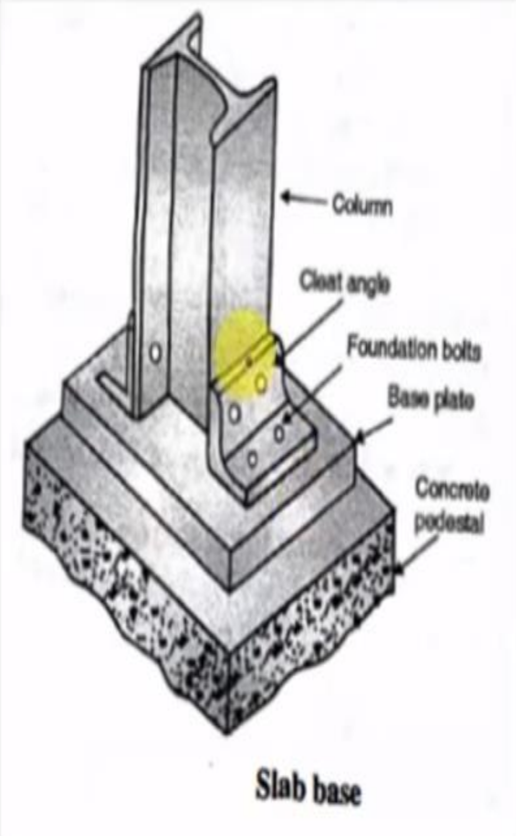

Isometric view

# Design of Column Bases

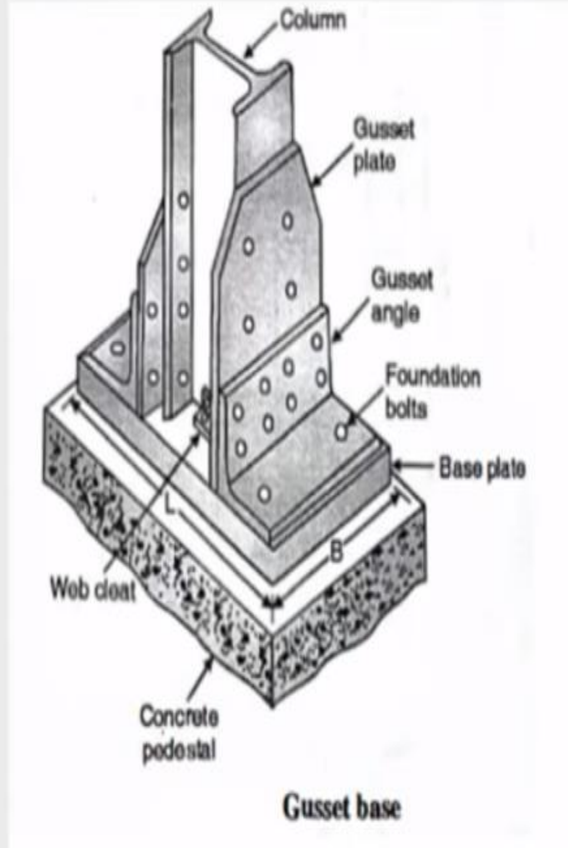
- Column bases transmit the column load to the concrete or masonry foundation blocks. ( The column base spread the load on wider area )
- The design compressive stress in a concrete footing is much smaller than it is in a steel column. So it becomes necessary that a suitable base plate should be provided below the column to distribute the load from it evenly to the footing below.
- The main function of the base plate is to spread the column load over a sufficiently wider area and keep the footing from being over stressed.

There are two types of column bases commonly used in practice.

| Slab Base | Gusseted Base |
| --- | --- |
|  |  |



Slab base



Gusset base

- Slab base are used in columns carrying small loads. In this type, the column is directly connected to the base plate through cleat angles
- The load is transferred to the base plate through bearing.

- For columns carrying heavy loads gusseted bases are used. In gusseted base, the column is connected to base plate through gusset.
- The load is transferred to the base Partly through bearing and partly through gusset.

Welding can be done in place of using cleat Angle.

## Design of Slab Base

### Design Procedure For Slab base ( Base Plate ) [ Page 46,47 ]

1) Assume a suitable grade of concert
2) Note down the properties of sections from steel table ( tf, tw, h, bf )
3) Calculate Bearing strength of concrete = 0.45 $f_{ck}$

   ( Example For M20 grade, Bearing Strength => 0.45 x 20 = 9 N/$mm^2$ )
4) Required area of a base plate (A) = $\frac{\text{Factored Load P}}{\text{Bearing strength of concrete}}$

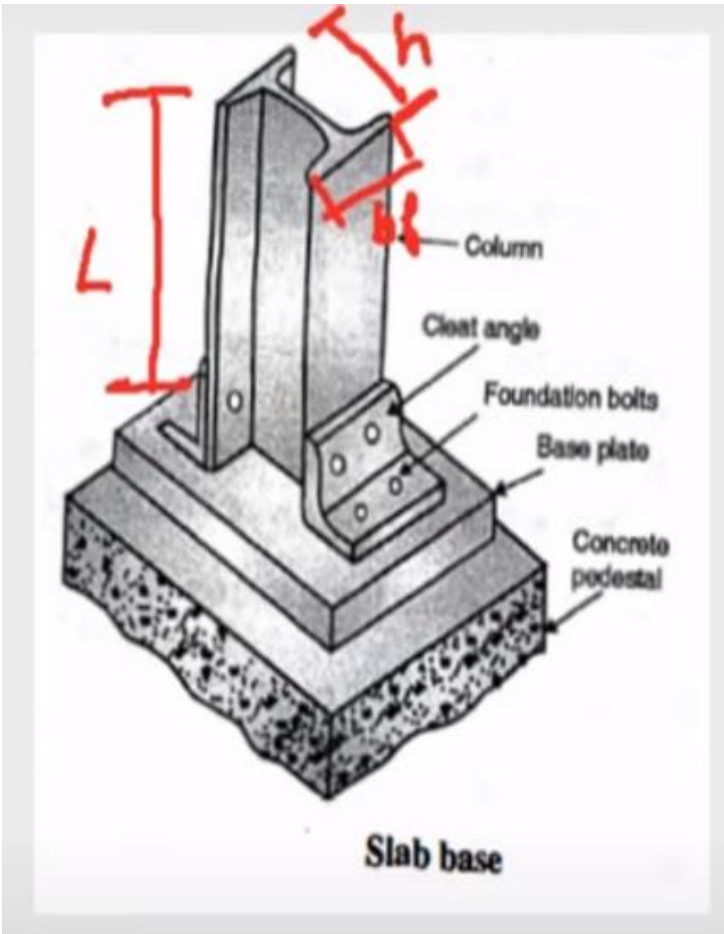
Now to Find Length and Width of Base plate, A = L x B

| L = h + 2a | B = bf + 2b | A= (h + 2a) x (bf + 2b) |
|---|---|---|

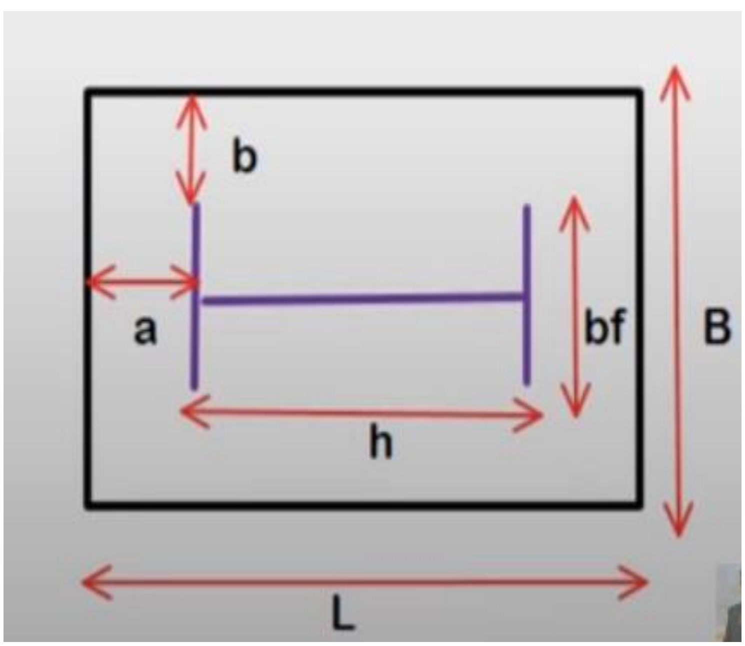
Assume a = b, For calculating unknown

5) Calculate area to be provided A' = L x B
6) The intensity of pressure (W) for the concrete pedestal (W) = $\frac{P}{A'} < 0.45\,fck$
7) Calculate thickness of slab (base plate)

$$ts = \sqrt{2.5 \times W \times (a^2 - 0.3b^2) \times \frac{\gamma_{mo}}{fy}} > tf$$


h
L
bf
Column
Cleat angle
Foundation bolts
Base plate
Concrete pedestal

Slab base


b
a
bf
B
h
L


W= upward Resisting pressure.

## Problem 1 Design of Slab Base

Design a slab base for a column ISHB 350 @ 67.4 kg/m carrying a factored load of 1000 KN. Also design the welded connection between slab base and column. Use concrete grade M20 and steel of Fe 410.

given

ISMB 350 @ 67.4

$P = 1000\ kN$

$P = 1000 \times 10^3\ N$

M20

$f_y = 250\ N/mm^2$

ISMB 350 @ 67.4

| | | | |
|---|---|---|---|
| | | | $t_f = 11.6\ mm$ |
| $h = 350\ mm$ | $b_f = 250\ mm$ | | $t_w = 8.3\ mm$ |

- Bearing strength of concrete $= 0.45\ f_{ck}$

  $\Rightarrow 0.45 \times 20 = 9\ N/mm^2$

- Required area of a base plate (A) = $\dfrac{\text{Factored Load } P}{\text{Bearing strength of concrete}}$

  $A = \dfrac{1000 \times 10^3}{9}$ $\quad A = 111111.1\ mm^2$

  $A = 11.11 \times 10^4\ mm^2$

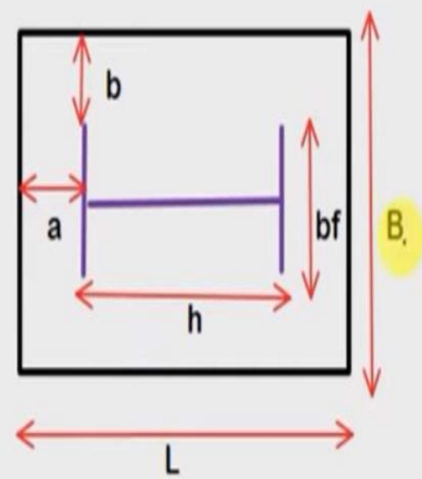
Now to Find Length and Width of Base plate, A = L x B

| | | |
|---|---|---|
| $L = h + 2a$ | $B = bf + 2b$ | $A = (h + 2a) \times (bf + 2b)$ |

Assume a = b, For calculating unknown

$11.11 \times 10^4 = (350 + 2a) \times (250 + 2b)$

$a = b = 18.52\ mm \approx 20\ mm$




$L = 350 + 2 \times 20 = $ $L = 390\ mm$

$B = 250 + 2 \times 20 = $ $B = 290\ mm$

$B = 250 + 2 \times 20 = $ $\boxed{B = 290\ mm}$

- Area to be provided $A' = L \times B$ $A' = 390 \times 290$ $A' = 113100$

$\boxed{A' = 11.31 \times 10^4\ mm^2}$

- The intensity of pressure (W) for the concrete pedestal (W) = $\frac{P}{A'} < 0.45\ fck$

$$W = \frac{1000 \times 10^3}{11.31 \times 10^4} = \boxed{W = 8.84\ N/mm^2} < 9$$

- Calculate thickness of slab (base plate)

$$ts = \sqrt{2.5 \times W \times (a^2 - 0.3b^2) \times \frac{r_{mo}}{fy}} > tf$$

$\gamma_{mo} = 1.1$

$f_y = 250\ mpa$

Fe410

$$ts = \sqrt{2.5 \times 8.84 \times (20^2 - 0.3 \times 20^2) \times \frac{1.1}{250}}$$

$ts = 5.21 \approx 6\ mm$

$tf = 11.6 \approx 12\ mm$

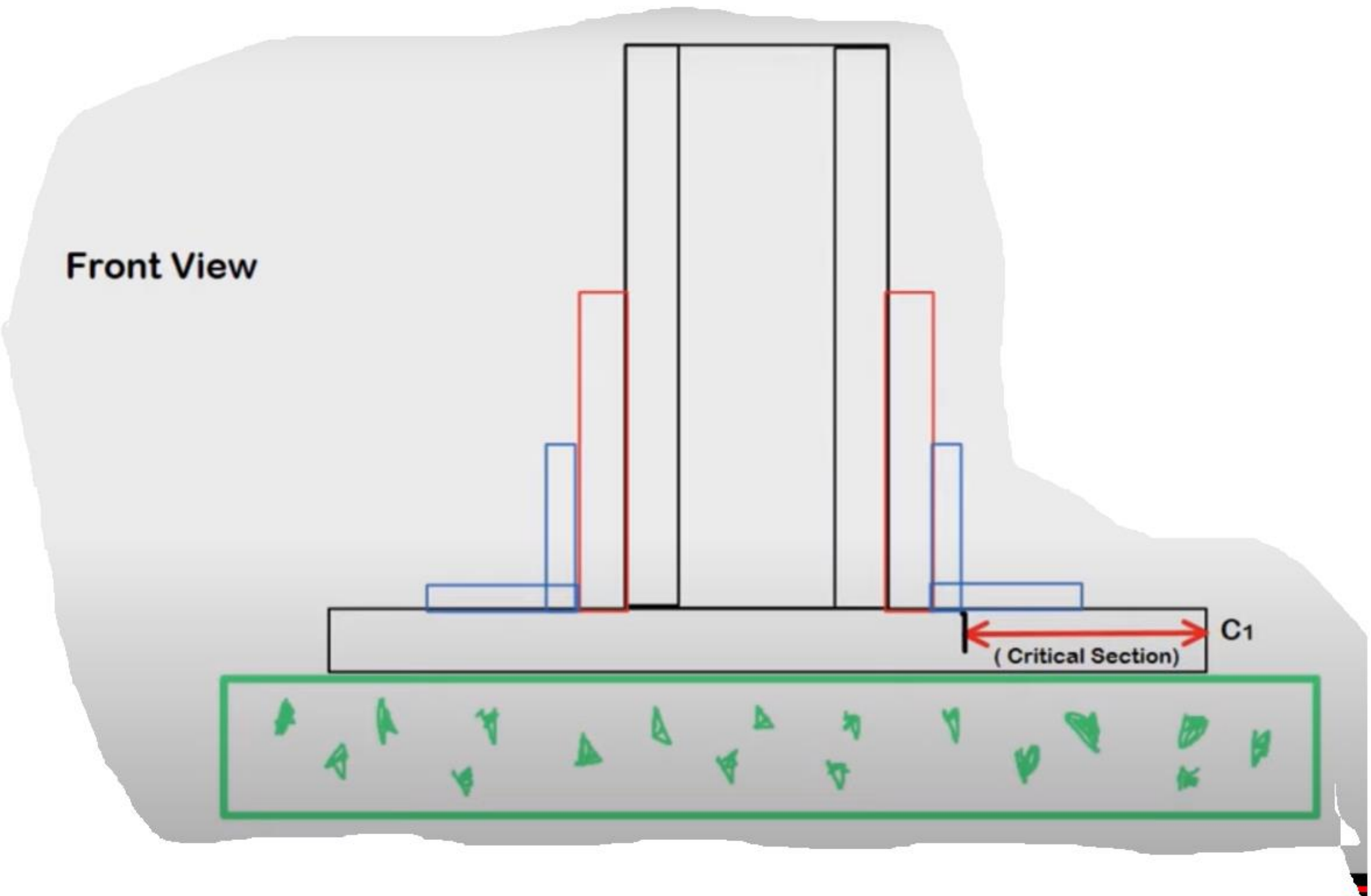
$ts > tf$ ✗

$ts < tf$ ✓ as ts

$\boxed{ts = 12\ mm}$

Provide 390 x 290 x 12 mm Base plate

GUSSET BASE



# CLEAT ANGLE

1. connects the flanges of the column to the base plate. In addition to these, web cleats are provided to connect the web of the column to the base plate and to guard against the possible dislocation of the column during erection.
2. cleats are used to prevent premature failing of the base connections.
3.